Supplemental Material

Design of Double Pipe Heat Exchanger Structures using Linear Models

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Section S1

This section depicts the constraints and objective function for the MINLP proposed by Peccini et al, 2019 [1]. For further explanations on the proposed model refer to the original article.

Discrete Representation of Geometric and Structural Variables. The hairpin dimensions are usually selected from a set of discrete values according to the available standard options. This design feature imposes the following relations involving the sets of binary variables that represent the available options:

$$dte = \sum_{sd=1}^{sdmax} \widehat{pdte}_{sd} yd_{sd} \quad \text{(S1)}$$

$$dti = \sum_{sd=1}^{sdmax} \widehat{pdti}_{sd} yd_{sd} \quad \text{(S2)}$$

$$Dte = \sum_{sD=1}^{sDmax} \widehat{pDte}_{sD} yD_{sD} \quad \text{(S3)}$$

$$Dti = \sum_{sD=1}^{sDmax} \widehat{pDti}_{sD} yD_{sD} \quad \text{(S4)}$$

$$Lh = \sum_{sL_h=1}^{sL_hmax} \widehat{pLh}_{sLh} yLh_{sLh} \quad \text{(S5)}$$

$$\sum_{sd=1}^{sdmax} yd_{sd} = 1 \quad \text{(S6)}$$

$$\sum_{sD=1}^{sDmax} yD_{sD} = 1 \quad \text{(S7)}$$

$$\sum_{sLh=1}^{sLhmax} yLh_{sLh} = 1 \quad \text{(S8)}$$

where dte and dti are the outer and inner diameters of the inner tube, Dte and Dti are the outer and inner diameters of the outer tube, and Lh is the tube length of each hairpin. The corresponding binary variables which indicate the discrete options selected are yd_{sd} for the inner tube diameter (discrete values: $\widehat{pdte}_{sd}$ and $\widehat{pdti}_{sd}$), yD_{sD} for the outer tube diameter (discrete values: $\widehat{pDte}_{sD}$ and $\widehat{pDti}_{sD}$), and yLh_{sLh} for the tube length (discrete values: $\widehat{pLh}_{sLh}$).

The selection of the number of parallel branches present in the heat exchanger design (*NB*), the number of units aligned in parallel in each branch for the tube-side stream (*NPt*) and for the annulus-side stream (*NPa*) as well as the number of countercurrent hairpins per unit (Nh) are also represented by binary variables (yB_{sB}, yPt_{sE}, yPa_{sE} and yNh_{sNh}, respectively).

$$NB = \sum_{sB=1}^{sBmax} \widehat{pNB}_{sB}\, yB_{sB} \tag{S9}$$

$$NPt = \sum_{sE=1}^{sEmax} \widehat{pNE}_{sE}\, yPt_{sE} \tag{S10}$$

$$NPa = \sum_{sE=1}^{sEmax} \widehat{pNE}_{sE}\, yPa_{sE} \tag{S11}$$

$$Nh = \sum_{sNh=1}^{sNhmax} \widehat{pNh}_{sNh} yNh_{sNh} \tag{S12}$$

$$\sum_{sB=1}^{sBmax} yB_{sB} = 1 \tag{S13}$$

$$\sum_{sE=1}^{sEmax} yPt_{sE} = 1 \tag{S14}$$

$$\sum_{sE=1}^{sEmax} yPa_{sE} = 1 \tag{S15}$$

$$\sum_{sNh=1}^{sNhmax} yNh_{sNh} = 1 \tag{S16}$$

where $\widehat{pNB}_{sB}$, is the set of discrete values of the number of parallel branches (1, 2,…, *sBmax*), $\widehat{pNE}_{sE}$ is the sequence of integer numbers representing the possible numbers of units interconnected along a branch (1, 2, …, sE_{max}) and $\widehat{pNh}_{sNh}$ is the sequence of integer numbers representing the number of hairpins possible per unit (1, 2, …, sNh_{max}).

Additional constraints must be included to ensure the selection of proper dimensions and structures:

$$yPt_{sE=1} + yPa_{sE=1} \geq 1 \tag{S17}$$

$$yd_{sd} + yD_{sD} \leq 1 \qquad \forall\ (sd, sD) \in SDD \tag{S18}$$

where SDD is a set of forbidden combinations (sd,sD).

Stream Allocation. The stream allocation is controlled by the binary variables yT_c and yT_h. If $yT_c = 1$, then the cold stream flows inside the inner tube and the hot stream flows in the annulus; if $yT_h = 1$, then the it is the other way around. The following equations relate the mass flow rates, physical properties and fouling factors of the hot and cold streams to the corresponding values of the tube-side and annulus-side flows:

$$mt = \widehat{m_c}\ yT_c + \widehat{m_h}\ yT_h \tag{S19}$$

$$ma = \widehat{m_c}\ yT_h + \widehat{m_h}\ yT_c \tag{S20}$$

$$\rho t = \hat{\rho}_c\ yT_c + \hat{\rho}_h\ yT_h \tag{S21}$$

$$\rho a = \hat{\rho}_c\ yT_h + \hat{\rho}_h\ yT_c \tag{S22}$$

$$Cpt = \widehat{Cp}_c\ yT_c + \widehat{Cp}_h\ yT_h \tag{S23}$$

$$Cpa = \widehat{Cp}_c\ yT_h + \widehat{Cp}_h\ yT_c \tag{S24}$$

$$\mu t = \hat{\mu}_c\ yT_c + \hat{\mu}_h\ yT_h \tag{S25}$$

$$\mu a = \hat{\mu}_c\ yT_h + \hat{\mu}_h\ yT_c \tag{S26}$$

$$kt = \hat{k}_c\ yT_c + \hat{k}_h\ yT_h \tag{S27}$$

$$ka = \hat{k}_c\ yT_h + \hat{k}_h\ yT_c \tag{S28}$$

$$Rft = \widehat{Rf}_c\ yT_c + \widehat{Rf}_h\ yT_h \tag{S29}$$

$$Rfa = \widehat{Rf_c}\, yT_h + \widehat{Rf_h}\, yT_c \tag{S30}$$

$$yT_c + yT_h = 1 \tag{S31}$$

where *m* is the mass flow rate, ρ is the density, *Cp* is the heat capacity, μ is the viscosity, *k* is the thermal conductivity, *Rf* is the fouling factor, and the subscripts *c* and *h* indicate the cold and hot streams, respectively.

Structural Constraints. The flow area and length of the hydraulic path of the streams considering the double pipe heat exchanger structure depend on the tube diameters, length, the number of hairpins and units in series as well as in parallel in each branch, and the number of parallel branches. The equations are:

$$At = \left(\frac{\pi dti^2}{4}\right) NB\ \ NPt \tag{S32}$$

$$Aa = \left(\frac{\pi Dti^2}{4} - \frac{\pi dte^2}{4}\right) NB\ \ NPa \tag{S33}$$

$$Lu = Nh\, Lh \tag{S34}$$

$$Lt = Lu\, NPa \tag{S35}$$

$$La = Lu\, NPt \tag{S36}$$

where *At* and *Aa* are the tube-side and annulus-side flow area, *Lu* is the total length of one unit and *Lt* and *La* are the corresponding flow path lengths for the streams flowing in the inner tube and in the annulus, respectively.

Inner tube Side Thermal and Hydraulic Modeling. Ignoring the minor head losses in the connections and bends, the pressure drop of the flow in the inner tube is calculated by the Darcy-Weisbach equation (omitting the viscosity correction factor) [12]:

$$\Delta Pt = \rho t\, ft \frac{Lt}{dti} \frac{vt^2}{2} \tag{S37}$$

where ft is the Darcy friction factor. The friction factor depends on the flow regime as follows[15]:

$$ft^{lam} = \frac{64}{Ret} \qquad \text{for } Ret \leq 1311 \qquad \text{(S38)}$$

$$ft^{tran} = 0.0488 \qquad \text{for } 1311 < Ret \leq 3380 \qquad \text{(S39)}$$

$$ft^{turb} = 0.014 + \frac{1.056}{Ret^{0{,}42}} \qquad \text{for } Ret > 3380 \qquad \text{(S40)}$$

where the Reynolds number is given by $Ret = (dti\ vt\ \rho t)/\mu t$ and in turn the velocity given by $vt = (mt/\rho t)/At$. For the transitional and turbulent flow, the Gnielinski correlation gives the Nusselt number [14]:

$$Nut^{Gni} = \frac{(ft/8)\ (Ret - 1000)\ Prt}{1 + 12.7\ (ft/8)^{1/2}\ (Prt^{2/3} - 1)} \qquad \text{for } Ret > 2300 \qquad \text{(S41)}$$

where Prandtl number of inner stream is given by: $Prt = Cpt\ \mu t/kt$. Because the laminar flow is affected by the entry region, more than one equation is utilized, according to Incropera et al [14]. For $Prt > 5$, the Hausen correlation is used:

$$Nut^{Hau} = 3.66 + \frac{0.0668\ (2\ dti/Lh)\ Ret\ Prt}{1 + 0.04((2\ dti/Lh)Ret\ Prt)^{2/3}} \qquad \text{for } Ret \leq 2300 \text{ and } Prt > 5 \qquad \text{(S42)}$$

For $Prt \leq 5$, the Nusselt number is specified by the Sieder & Tate correlation unless its given value is lower than the theoretical Nusselt number for fully developed flow (3.66), in which case the latter is applied:

$$Nut^{S\&T} = 1.86 \left(\frac{Ret\ Prt\ dti}{Lh/2}\right)^{\frac{1}{3}} \qquad \text{for } Ret \leq 2300, Prt \leq 5, Nut^{S\&T} \geq 3.66 \qquad \text{(S43)}$$

$$Nut^{theo} = 3.66 \qquad \text{for } Ret \leq 2300, Prt \leq 5, Nut^{S\&T} < 3.66 \qquad \text{(S44)}$$

Without loss of generality, the viscosity correction factor in the Sieder & Tate correlation in Equation (S43) has been omitted (i.e. the bulk and wall viscosities are considered equal). The following equations relate binary variables and the corresponding ranges for the *Re*, *Pr*, and *Nu* numbers :

$$Ret \leq 1311\, yRet_1 + 2300\, yRet_2 + 3380\, yRet_3 + \widehat{URe}\, yRet_4 \quad \text{(S45)}$$

$$Ret \geq 1311 yRet_2 + 2300 yRet_3 + 3380\, yRet_4 + \hat{\varepsilon} \quad \text{(S46)}$$

$$Prt \leq 5 yPrt_1 + \widehat{UPr}\, yPrt_2 \quad \text{(S47)}$$

$$Prt \geq 5 yPrt_2 + \hat{\varepsilon} \quad \text{(S48)}$$

$$Nut^{S\&T} \leq 3.66 yNut_1 + \widehat{UNu}\, yNut_2 - \hat{\varepsilon} \quad \text{(S49)}$$

$$Nut^{S\&T} \geq 3.66 yNut_2 \quad \text{(S50)}$$

where $\hat{\varepsilon}$ is a small positive number. Since only one binary variable must be selected for each set of intervals, it yields:

$$\sum_{sRet=1}^{sRetmax} yRet_{sRet} = 1 \quad \text{(S51)}$$

$$yPrt_1 + yPrt_2 = 1 \quad \text{(S52)}$$

$$yNut_1 + yNut_2 = 1 \quad \text{(S53)}$$

Therefore, the friction factor and the Nusselt number are represented by:

$$ft = ft^{lam} yRet_1 + ft^{tran}\,(yRet_2 + yRet_3) + ft^{turb} yRet_4 \quad \text{(S54)}$$

$$\begin{aligned} Nut = {} & Nut^{theo}(yRet_1 + yRet_2) yPrt_1 yNut_1 \\ & + Nut^{S\&T}(yRet_1 + yRet_2) yPrt_1 yNut_2 \\ & + Nut^{Hau}(yRet_1 + yRet_2) yPrt_2 + Nut^{Gni}(yRet_3 + yRet_4) \end{aligned} \quad \text{(S55)}$$

Annulus Side Thermal and Hydraulic Modeling. Ignoring the head losses in the connections and bends, the pressure drop of the flow in the annulus is given by the Darcy-Weisbach equation using the hydraulic diameter (also omitting the viscosity correction factor) [12]:

$$\Delta Pa = \rho a\, fa \frac{Lt}{dh}\frac{va^2}{2} \tag{S56}$$

where *fa* is the Darcy friction factor for the annular flow. The annular region friction factor, analogously to the inner tube, depends on the flow regime according to the following equations [14]:

$$fa^{lam} = \frac{64}{Rea} \qquad \text{for } Rea \leq 500 \tag{S57}$$

$$fa^{tran} = 0.02696 + \frac{32.656}{Rea^{0.93}} \qquad \text{for } 500 < Rea \leq 10000 \tag{S58}$$

$$fa^{turb} = \frac{0.178}{Rea^{0.1865}} \qquad \text{for } Rea > 10000 \tag{S59}$$

The Reynolds number is $Rea = (dh\ va\ \rho a)/\mu a$ (velocity given by $va = (ma/\rho a)/Aa$). The hydraulic diameter is here calculated as four times the flow cross-sectional area divided by the wetted perimeter, which in the case of only one concentric tube can be simplified to [14]:

$$dh = Dti - dte \tag{S60}$$

Regarding the Nusselt number, the same ranges and correlations used for the tube-side are applied, replacing the inner tube diameter by the hydraulic diameter:

$$Nua^{theo} = 3.66 \qquad \text{for } Rea \leq 2300, Pra \leq 5 \text{ and } Nua^{S\&T} < 3.66 \tag{S61}$$

$$Nua^{S\&T} = 1.86\left(\frac{Rea\ Pra\ dh}{Lh/2}\right)^{\frac{1}{3}} \qquad \text{for } Rea \leq 2300, Pra \leq 5 \text{ and } Nua^{S\&T} \geq 3.66 \tag{S62}$$

$$Nua^{Hau} = 3.66 + \frac{0.0668\ (2\ dh/Lh)Rea\ Pra}{1 + 0.04\left((2\ dh/Lh)Rea\ Pra\right)^{\frac{2}{3}}} \qquad \text{for } Rea \leq 2300 \text{ and } Pra > 5 \tag{S63}$$

$$Nua^{Gni} = \frac{(fa/8)(Rea - 1000)Pra}{1 + 12.7\ (fa/8)^{\frac{1}{2}}\ (Pra^{\frac{2}{3}} - 1)} \qquad \text{for } Rea > 2300 \tag{S64}$$

where the Prandtl number is given by $Pra = Cpa\frac{\mu a}{ka}$. Binary variables are then included to describe each interval of Rea, Pra and $Nua^{S\&T}$:

$$Rea \ \leq 500\ yRea_1 + 2300\ yRea_2 + 10000\ yRea_3 + \widehat{URe}\ yRea_4 \tag{S65}$$

$$Rea \ \geq 500yRea_2 + 2300yRea_3 + 10000\ yRea_4 + \varepsilon \tag{S66}$$

$$Pra \ \leq 5yPra_1 + \widehat{UPr}\ yPra_2 \tag{S67}$$

$$Pra \ \geq 5yPra_2 + \varepsilon \tag{S68}$$

$$Nua^{S\&T} \leq 3.66yNua_1 + \widehat{UNu}\ yNua_2 - \varepsilon \tag{S69}$$

$$Nua^{S\&T} \geq 3.66yNua_2 \tag{S70}$$

$$\sum_{sRea=1}^{sReamax} yRea_{sRea} = 1 \tag{S71}$$

$$yPra_1 + yPra_2 = 1 \tag{S72}$$

$$yNua_1 + yNua_2 = 1 \tag{S73}$$

$$fa = fa^{lam}yRea_1 + fa^{tran}\ (yRea_2 + yRea_3) + fa^{turb}yRea_4 \tag{S74}$$

$$\begin{aligned} Nua = Nua^{theo}&(yRea_1 + yRea_2)yPra_1yNua_1 \\ &+ Nua^{S\&T}(yRea_1 + yRea_2)yPra_1yNua_2 \\ &+ Nua^{Hau}(yRea_1 + yRea_2)yPra_2 \\ &+ Nua^{Gni}(yRea_3 + yRea_4) \end{aligned} \tag{S75}$$

Heat Transfer Coefficients. The overall heat transfer coefficient is then determined by:

$$U = \frac{1}{\frac{1}{ht}\frac{dte}{dti} + Rft\frac{dte}{dti} + \frac{dte\ln\left(\frac{dte}{dti}\right)}{2\widehat{ktube}} + Rfa + \frac{1}{ha}} \tag{S76}$$

where $\widehat{ktube}$ is the thermal conductivity of the inner tube and the convective heat transfer coefficients ht and ha are obtained from the respective Nusselt numbers $ht = Nut\ kt/dti$ and $ha = Nua\frac{ka}{dh}$.

Heat Transfer Rate. According to the LMTD method, the heat transfer rate is given by:

$$\hat{Q} = UA_{req}\widehat{\Delta Tlm}\,F \tag{S77}$$

where A_{req} is the required heat transfer area, $\widehat{\Delta Tlm}$ is the logarithmic mean temperature, and F is the correction factor. In turn, the logarithmic mean temperature is defined as:

$$\widehat{\Delta Tlm} = \frac{\left(\widehat{Ti}_h - \widehat{To}_c\right) - \left(\widehat{To}_h - \widehat{Ti}_c\right)}{\ln\left(\frac{\left(\widehat{Ti}_h - \widehat{To}_c\right)}{\left(\widehat{To}_h - \widehat{Ti}_c\right)}\right)} \tag{S78}$$

where $\widehat{Ti}_h$ and $\widehat{To}_h$ are the inlet and outlet temperatures of the hot stream, and $\widehat{Ti}_c$ and $\widehat{To}_c$ are the inlet and outlet temperatures of the cold stream. For stream sST (cold, $sSt = c$, or hot, $sSt = h$) aligned in series and the other stream in parallel, the correction factor is given by [13]:

$$\widehat{pF}_{sST,sE} = \frac{\left(\widehat{pR}_{sST} - \widehat{pNE}_{sE}\right)}{\widehat{pNE}_{sE}\left(\widehat{pR}_{sST} - 1\right)} \frac{\ln\left(\frac{1 - \widehat{pP}_{sST}}{1 - \widehat{pP}_{sST}\widehat{pR}_{sST}}\right)}{\ln\left(\frac{\widehat{pR}_{sST} - \widehat{pNE}_{sE}}{\widehat{pR}_{sST}\left(1 - \widehat{pP}_{sST}\widehat{pR}_{sST}\right)^{1/\widehat{pNE}_{sE}}} + \frac{\widehat{pNE}_{sE}}{\widehat{pR}_{sST}}\right)} \quad \text{for } sE \neq 1 \tag{S79}$$

where the factors $\widehat{pR}_{sST}$ and $\widehat{pP}_{sST}$ are specified as:

$$\widehat{pR}_{sSt} = \begin{cases} \frac{\left(\widehat{To}_c - \widehat{Ti}_c\right)}{\left(\widehat{Ti}_h - \widehat{To}_h\right)} & \text{for } sSt = c \\ \frac{\left(\widehat{Ti}_h - \widehat{To}_h\right)}{\left(\widehat{To}_c - \widehat{Ti}_c\right)} & \text{for } sSt = h \end{cases} \tag{S80}$$

$$\widehat{pP}_{sST} = \begin{cases} \frac{\left(\widehat{Ti}_h - \widehat{To}_h\right)}{\left(\widehat{Ti}_h - \widehat{Ti}_c\right)} & \text{for } sSt = c \\ \frac{\left(\widehat{To}_c - \widehat{Ti}_c\right)}{\left(\widehat{Ti}_h - \widehat{Ti}_c\right)} & \text{for } sSt = h \end{cases} \tag{S81}$$

Based on these expressions, the constraint that represents the correction factor (F) evaluation, dependent on the structure selected, becomes:

$$F = 1 + \sum_{sE=2}^{sEmax} \sum_{sE'=2}^{sEmax} \{yT_c[yPt_{sE}(\widehat{pF}_{h,sE} - 1) + yPa_{sE'}(\widehat{pF}_{c,sE'} - 1)] + yT_h[yPt_{sE}(\widehat{pF}_{c,sE} - 1) + yPa_{sE'}(\widehat{pF}_{h,sE'} - 1)]\} \quad \text{(S82)}$$

In turn, aiming at guaranteeing a design margin, a minimum area excess ($\hat{A}_{exc}$) is imposed.

$$A \geq \left(1 + \frac{\hat{A}_{exc}}{100}\right) A_{req} \quad \text{(S83)}$$

where the equipment heat transfer area is given by:

$$A = \pi\ dte\ Lu\ NB\ NPt\ NPa \quad \text{(S84)}$$

Therefore, the heat transfer rate shown in equation (S77) can be reorganized as:

$$UA \geq \frac{\hat{Q}\left(\frac{\widehat{Aexc}}{100} + 1\right)}{\widehat{\Delta Tlm}\ \ F} \quad \text{(S85)}$$

Pressure drop and velocity bounds. The lower and upper bounds on the velocities of the tube-side flow and annulus-side flow are given by:

$$vt \geq \widehat{vt}_{min} \quad \text{(S86)}$$

$$vt \leq \widehat{vt}_{max} \quad \text{(S87)}$$

$$va \geq \widehat{va}_{min} \quad \text{(S88)}$$

$$va \leq \widehat{va}_{max} \quad \text{(S89)}$$

While the pressure drop bounds are represented by:

$$\Delta Pt \leq \widehat{\Delta P}_{c_{disp}} yT_c + \widehat{\Delta P}_{h_{disp}} yT_h \quad \text{(S90)}$$

$$\Delta Pa \leq \widehat{\Delta P}_{c_{disp}} yT_h + \widehat{\Delta P}_{h_{disp}} yT_c \tag{S91}$$

Objective function. The objective function is given by the minimization of the heat transfer area:

$$\min A + \widehat{ph} Nh \tag{S92}$$

where *ph* is a penalty factor (a random small value) associated to the number of hairpins directing the optimization, in case of equivalent solutions, to the one with smaller number of elements.

Section S2

Three alternatives of parameter and variables were tested to generate new parametrized MILM's with a reduced number of variables and constraint and thus enable the problem to be solved faster and with less memory requirements. They are called here Par-GEO-MILM, Par-FLU-MILM e Par-STR-MILM (the latter is the option that presented the best performance and was adopted in the paper).

This section contains the constraints and objective function for the three alternatives.

Par-GEO-MILM. For the first alternative, the geometric variables (*dte, Dte* and *Lh*) are transformed into parameters ($\widehat{pdte}$, $\widehat{pDte}$ and $\widehat{pLh}$) and hence the binary variables responsible for its discrete option selections (yd_{sd}, yD_{sD} and yLh_{sLh}) are no longer needed. The MINLP showed in Section S1 is rearranged and the same techniques (described in Costa and Bagajewicz [15]) employed for the generation of the original MILM presented in this study are applied, rendering the Par-GEO-MILM model shown in this section.

The selection of the number of parallel branches, the number of units aligned in parallel in each branch for the tube-side stream and for the annulus-side stream, as well as the number of countercurrent hairpins per unit present in the heat exchanger design are represented by the binary variables yB_{sB}, yPt_{sE}, yPa_{sE} and yNh_{sNh}, respectively:

$$\sum_{sB=1}^{sBmax} yB_{sB} = 1 \qquad \text{(S93)}$$

$$\sum_{sE=1}^{sEmax} yPt_{sE} = 1 \qquad \text{(S94)}$$

$$\sum_{sE=1}^{sEmax} yPa_{sE} = 1 \qquad \text{(S95)}$$

$$\sum_{sNh=1}^{sNhmax} yNh_{sNh} = 1 \qquad \text{(S96)}$$

$$yPt_{sE=1} + yPa_{sE=1} \geq 1 \qquad \text{(S97)}$$

The stream allocation is controlled by the binary variables yT_c and yT_h:

$$yT_c + yT_h = 1 \tag{S98}$$

The flow velocity inside the inner tube, its corresponding Reynolds number and additional linear inequalities are:

$$vt = \sum_{sB=1}^{sBmax} \sum_{sE=1}^{sEmax} \sum_{sST} \frac{4\,\widehat{m}_{sST}}{\pi \widehat{pdti}^2\ \hat{\rho}_{sST}\ \widehat{pNB}_{sB} \widehat{pNE}_{sE}}\ wvt_{sB,sE,sST} \tag{S99}$$

$$Ret = \sum_{sB=1}^{sBmax} \sum_{sE=1}^{sEmax} \sum_{sST} \frac{4\,\widehat{m}_{sST}}{\pi\ \widehat{pdti}\ \hat{\mu}_{sST}\ \widehat{pNB}_{sB} \widehat{pNE}_{sE}}\ wvt_{sB,sE,sST} \tag{S100}$$

$$wvt_{sB,sE,sST} \leq yB_{sB} \tag{S101}$$

$$wvt_{sB,sE,sST} \leq yPt_{sE} \tag{S102}$$

$$wvt_{sB,sE,sST} \leq yT_{sST} \tag{S103}$$

$$wvt_{sB,sE,sST} \geq yB_{sB} + yPt_{sE} + yT_{sST} - 2 \tag{S104}$$

The Prandtl number of the inner tube stream becomes:

$$Prt = \frac{\widehat{Cp}_c \hat{\mu}_c}{\hat{k}_c} yT_c + \frac{\widehat{Cp}_h \hat{\mu}_h}{\hat{k}_h}\ yT_h \tag{S105}$$

The evaluation of the Nusselt number by the Sieder & Tate correlation is now:

$$Nut^{S\&T} = \sum_{sB=1}^{sBmax} \sum_{sE=1}^{sEmax} \sum_{sST} \widehat{pNut}^{S\&T}_{sB,sE,sST} wNut_{sB,sE,sST} \tag{S106}$$

$$wNut_{sB,sE,sST} \leq yB_{sB} \tag{S107}$$

$$wNut_{sB,sE,sST} \leq yPt_{sE} \tag{S108}$$

$$wNut_{sB,sE,sST} \leq yT_{sST} \tag{S109}$$

$$wNut_{sB,sE,sST} \geq yB_{sB} + yPt_{sE} + yT_{sST} - 2 \tag{S110}$$

where the parameter $\widehat{pNut}^{S\&T}_{sB,sE,sST}$, is given by:

$$\widehat{pNut}^{S\&T}_{sB,sE,sST} = 1.86\left(\frac{8\ \widehat{Cp}_{sST}\ \widehat{m}_{sST}}{\pi\ \hat{k}_{sST}\ \widehat{pNB}_{sB}\ \widehat{pNE}_{sE}\ \widehat{pLh}}\right)^{1/3} \tag{S111}$$

The constraints relating binary variables to Re ranges for the tube-side remain the same:

$$Ret \ \leq 1311\ yRet_1 + 2300\ yRet_2 + 3380\ yRet_3 + \widehat{URe}\ yRet_4 \tag{S112}$$

$$Ret \ \geq 1311 yRet_2 + 2300 yRet_3 + 3380\ yRet_4 + \hat{\varepsilon} \tag{S113}$$

$$Prt \ \leq 5yPrt_1 + \widehat{UPr}\ yPrt_2 \tag{S114}$$

$$Prt \ \geq 5yPrt_2 + \hat{\varepsilon} \tag{S115}$$

$$Nut^{S\&T} \leq 3.66yNut_1 + \widehat{UNu}\ yNut_2 - \hat{\varepsilon} \tag{S116}$$

$$Nut^{S\&T} \geq 3.66yNut_2 \tag{S117}$$

$$\sum_{sRet=1}^{sRetmax} yRet_{sRet} = 1 \tag{S118}$$

$$yPrt_1 + yPrt_2 = 1 \tag{S119}$$

$$yNut_1 + yNut_2 = 1 \tag{S120}$$

The pressure drop in the tubes becomes:

$$\begin{aligned}\Delta Pt = \sum_{sB=1}^{sBmax}\sum_{sNh=1}^{sNhmax}\sum_{sE=1}^{sEmax}\sum_{sE'=1}^{sEmax}\sum_{sST}\Big(&\widehat{pdPt}1_{sB,sE,sNh,sE',sST}\,wdPt_{sB,sNh,sE,sE',sST,1}\\ &+ \widehat{pdPt}23_{sB,sE,sNh,sE',sST}\,wdPt_{sB,sNh,sE,sE',sST,2}\\ &+ \widehat{pdPt}23_{sB,sE,sNh,sE',sST}\,wdPt_{sB,sNh,sE,sE',sST,3}\\ &+ \widehat{pdPt}4_{sB,sE,sNh,sE',sST}\,wdPt_{sB,sNh,sE,sE',sST,4}\Big)\end{aligned} \tag{S121}$$

$$wdPt_{sB,sNh,sE,sE',sST,sRet} \leq wA_{sB,sNh,sE,sE'} \tag{S122}$$

$$wdPt_{sB,sNh,sE,sE',sST,sRet} \leq yT_{sST} \tag{S123}$$

$$wdPt_{sB,sNh,sE,sE',sST,sRet} \leq yRet_{sRet} \tag{S124}$$

$$wdPt_{sB,sNh,sE,sE',sST,sRet} \geq wA_{sB,sNh,sE,sE'} + yT_{sST} + yRet_{sRet} - 2 \tag{S125}$$

The additional parameters inserted for simplification purposes in Eq. (S121) are given by:

$$\widehat{pdPt}1_{sB,sE,sNh,sE',sST} = \frac{128\ \hat{\mu}_{sST}\ \widehat{m}_{sST}\widehat{pNh}_{sNh}\widehat{pLh}\widehat{pNE}_{sE'}}{\pi\ \hat{\rho}_{sST}\ \widehat{pdti}^4\widehat{pNB}_{sB}\widehat{pNE}_{sE}} \tag{S126}$$

$$\widehat{pdPt}23_{sB,sE,sNh,sE',sST} = \frac{0.3904\ \widehat{m}_{sST}^2\widehat{pNh}_{sNh}\widehat{pLh}\widehat{pNE}_{sE'}}{\pi^2\ \hat{\rho}_{sST}\widehat{pdti}^5\widehat{pNB}_{sB}^2\widehat{pNE}_{sE}^2} \tag{S127}$$

$$\begin{aligned}\widehat{pdPt}4_{sB,sE,sNh,sE',sST} &= \frac{0.112\ \widehat{m}_{sST}^2\widehat{pNh}_{sNh}\widehat{pLh}\widehat{pNE}_{sE'}}{\pi^2\ \hat{\rho}_{sST}\ \widehat{pdti}^5\widehat{pNB}_{sB}^2\widehat{pNE}_{sE}^2} \\ &+ \frac{4.719\ \hat{\mu}_{sST}^{0.42}\ \widehat{m}_{sST}^{1.58}\widehat{pNh}_{sNh}\widehat{pLh}\ \widehat{pNE}_{sE'}}{\pi^{1.58}\ \hat{\rho}_{sST}\ \widehat{pdti}^{4.58}\widehat{pNB}_{sB}^{1.58}\widehat{pNE}_{sE}^{1.58}}\end{aligned} \tag{S128}$$

The annulus-side stream velocity, its corresponding Reynolds number and additional linear inequalities are given by:

$$va = \sum_{sB=1}^{sBmax}\sum_{sE=1}^{sEmax}\sum_{sST}\frac{\widehat{m}_{sST^*}}{\hat{\rho}_{sST^*}\ \widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}wva_{sB,sE,sST} \tag{S129}$$

$$Rea = \sum_{sB=1}^{sBmax}\sum_{sE=1}^{sEmax}\sum_{sST}\frac{\widehat{m}_{sST^*}\ \widehat{pdh}}{\hat{\mu}_{sST^*}\ \widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}wva_{sB,sE,sST} \tag{S130}$$

$$wva_{sB,sE,sST} \le yB_{sB} \tag{S131}$$

$$wva_{sB,sE,sST} \le yPa_{sE} \tag{S132}$$

$$wva_{sB,sE,sST} \le yT_{sST} \tag{S133}$$

$$wva_{sB,sE,sST} \ge yB_{sB} + yPa_{sE} + yT_{sST} - 2 \tag{S134}$$

where if $sST = h$, then $sST^* = c$ and vice-versa.

The Prandtl number and the Nusselt number evaluation by the Sieder & Tate correlation for the annulus-side stream becomes:

$$Pra = \frac{\widehat{Cp}_c\hat{\mu}_c}{\hat{k}_c}\ yT_h + \frac{\widehat{Cp}_h\hat{\mu}_h}{\hat{k}_h}\ yT_c \tag{S135}$$

$$Nua^{S\&T} = \sum_{sB=1}^{sBmax}\sum_{sE=1}^{sEmax}\sum_{sST}\widehat{pNua}_{sB,sE,sST}^{S\&T}\ wva_{sB,sE,sST} \tag{S136}$$

where the parameter $\sum_{sST} \widehat{pNua}^{S\&T}_{sB,sE,sST}$ is given by:

$$\widehat{pNua}^{S\&T}_{sB,sE,sST} = 1.86 \left(\frac{2\, \widehat{Cp}_{sST^*}\, \widehat{m}_{sST^*}\, \widehat{pdh}^2}{\widehat{k}_{sST^*}\, \widehat{pAa}\, \widehat{pNB}_{sB} \widehat{pNE}_{sE} \widehat{pLh}} \right)^{\frac{1}{3}} \tag{S137}$$

where if $sST = h$, then $sST^* = c$ and vice-versa.

The constraints relating binary variables to *Re* ranges for the annular-side remain the same:

$$Rea \ \leq 500\, yRea_1 + 2300\, yRea_2 + 10000\, yRea_3 + \widehat{URe}\, yRea_4 \tag{S138}$$

$$Rea \ \geq 500 yRea_2 + 2300 yRea_3 + 10000\, yRea_4 + \varepsilon \tag{S139}$$

$$Pra \ \leq 5 yPra_1 + \widehat{UPr}\, yPra_2 \tag{S140}$$

$$Pra \ \geq 5 yPra_2 + \varepsilon \tag{S141}$$

$$Nua^{S\&T} \leq 3.66 yNua_1 + \widehat{UNu}\, yNua_2 - \varepsilon \tag{S142}$$

$$Nua^{S\&T} \geq 3.66 yNua_2 \tag{S143}$$

$$\sum_{sRea=1}^{sReamax} yRea_{sRea} = 1 \tag{S144}$$

$$yPra_1 + yPra_2 = 1 \tag{S145}$$

$$yNua_1 + yNua_2 = 1 \tag{S146}$$

The pressure drop of the flow in the annulus is given by:

$$\begin{aligned}\Delta Pa = \sum_{sB=1}^{sBmax} \sum_{sE=1}^{sEmax} \sum_{sNh=1}^{sNhmax} \sum_{sE'=1}^{sEmax} \sum_{sST} (&\widehat{pdPa}1_{sB,sE,sNh,sE',sST} wdPa_{sB,sNh,sE.sE' sST,1} \\ &+ \widehat{pdPa}23_{sB,sE,sNh,sE',sST} (wdPa_{sB,sNh,sE.sE' sST,sRea} \\ &+ wdPa_{sB,sNh,sE.sE' sST,3}) \\ &+ \widehat{pdPa}4_{sB,sE,sNh,sE',sST} wdPa_{sB,sNh,sE.sE' sST,4})\end{aligned} \tag{S147}$$

$$wdPa_{sB,sNh,sE.sE' sST,sRea} \leq wA_{sB,sNh,sE,sE'} \tag{S148}$$

$$wdPa_{sB,sNh,sE.sE'sST,sRea} \leq yT_{sST} \tag{S149}$$

$$wdPa_{sB,sNh,sE.sE'sST,sRea} \leq yRea_{sRea} \tag{S150}$$

$$wdPa_{sB,sNh,sE.sE'sST,sRea} \geq wA_{sB,sNh,sE,sE'} + yT_{sST} + yRea_{sRea} - 2 \tag{S151}$$

$$\widehat{pdPa}1_{sB,sE,sNh,sE',sST} = \frac{32\ \hat{\mu}_{sST^*}\ \hat{m}_{sST^*}\ \widehat{pNh}_{sNh}\widehat{pLh}\widehat{pNE}_{sE}}{\hat{\rho}_{sST^*}\ \widehat{pdh}^2\ \widehat{pAa}\widehat{pNB}_{sB}\ \widehat{pNE}_{sE'}} \tag{S152}$$

$$\widehat{pdPa}23_{sB,sE,sNh,sE',sST} = \frac{0.01348\ \hat{m}_{sST^*}^2\ \widehat{pNh}_{sNh}\widehat{pLh}\widehat{pNE}_{sE}}{\hat{\rho}_{sST^*}\ \widehat{pdh}\ \widehat{pAa}^2\widehat{pNB}_{sB}^2\widehat{pNE}_{sE'}^2} + \frac{16.328\ \hat{\mu}_{sST^*}^{0.93}\ \hat{m}_{sST^*}^{1.07}\ \widehat{pNh}_{sNh}\widehat{pLh}\widehat{pNE}_{sE}}{\hat{\rho}_{sST^*}\ \widehat{pdh}^{1.93}\ \widehat{pAa}^{1.07}\widehat{pNB}_{sB}^{1.07}\widehat{pNE}_{sE'}^{1.07}} \tag{S153}$$

$$\widehat{pdPa}4_{sB,sE,sNh,sE',sST} = \frac{0.089\ \ \hat{\mu}_{sST^*}^{0.1865}\ \hat{m}_{sST^*}^{1.8135}\ \widehat{pNh}_{sNh}\widehat{pLh}_{sLh}\widehat{pNE}_{sE}}{\hat{\rho}_{sST^*}\ \widehat{pdh}^{1.1865}\ \widehat{pAa}^{1.8135}\widehat{pNB}_{sB}^{1.8135}\widehat{pNE}_{sE'}^{1.8135}} \tag{S154}$$

where if $sST = h$, then $sST^* = c$ and vice-versa.

The equipment heat transfer area is given by:

$$A = \sum_{sB=1}^{sBmax}\sum_{sNh=1}^{sNhmax}\sum_{sE=1}^{sEmax}\sum_{sE'=1}^{sE'max} \widehat{pA}_{sB,sNh,sE,sE'}\ wA_{sB,sNh,sE,sE'} \tag{S155}$$

$$wA_{sB,sNh,sE,sE'} \leq yB_{sB} \tag{S156}$$

$$wA_{sB,sNh,sE,sE'} \leq yNh_{sNh} \tag{S157}$$

$$wA_{sB,sNh,sE,sE'} \leq yPt_{sE} \tag{S158}$$

$$wA_{sB,sNh,sE,sE'} \leq yPa_{sE'} \tag{S159}$$

$$wA_{sB,sNh,sE,sE'} \geq yB_{sB} + yNh_{sNh} + yPt_{sE} + yPa_{sE'} - 3 \tag{S160}$$

$$\widehat{pA}_{sB,sNh,sE,sE'} = \pi\ \widehat{pdte}\ \widehat{pNB}_{sB}\ \widehat{pNh}_{sNh}\widehat{pLh}\widehat{pNE}_{sE}\widehat{pNE}_{sE'} \tag{S161}$$

The heat transfer rate, based on the LMTD method, after reformulation becomes:

$$
\begin{aligned}
&\hat{Q}\sum_{sB=1}^{sBmax}\sum_{sE=1}^{sEmax}\sum_{sE'=1}^{sEmax}\sum_{sST}\Bigg[\sum_{sRet=1}^{2}\Bigg(\frac{\widehat{pdte}}{\hat{k}_{sST}\,\widehat{pNut}^{theo}}wht^{theo}_{sST,sRet,1,1}\\
&+\frac{\widehat{pdte}}{\hat{k}_{sST}\,\widehat{pNut}^{S\&T}_{sB,sE,sST}}wht^{S\&T}_{sB,sE,sST,sRet,1,2}+\frac{\widehat{pdte}}{\hat{k}_{sST}\,\widehat{pNut}^{Hau}_{sB,sE,sST}}wht^{Hau}_{sB,sE,sST,sRet,2}\Bigg)\\
&+\frac{\widehat{pdte}}{\hat{k}_{sST}\,\widehat{pNut}^{Gni}_{tran_{sB,sE,sST}}}wht^{Gni}_{sB,sE,sST,3}+\frac{\widehat{pdte}}{\hat{k}_{sST}\,\widehat{pNut}^{Gni}_{turb_{sB,sE,sST}}}wht^{Gni}_{sB,sE,sST,4}+\widehat{Rf}_{sST}\frac{\widehat{pdte}}{\widehat{pdti}}yT_{sST}\\
&+\frac{\widehat{pdte}\ln\left(\frac{\widehat{pdte}}{\widehat{pdti}}\right)}{2\widehat{ktube}}+\widehat{Rf}_{sST^*}yT_{sST}\\
&+\sum_{sRea=1}^{2}\Bigg(\frac{\widehat{pdh}}{\hat{k}_{sST^*}\,\widehat{pNua}^{theo}}wha^{theo}_{sST,sRea,1,1}+\frac{\widehat{pdh}}{\hat{k}_{sST^*}\,\widehat{pNua}^{S\&T}_{sB,sE',sST}}wha^{S\&T}_{sB,sE',sST,sRea,1,2}\\
&+\frac{\widehat{pdh}}{\hat{k}_{sST^*}\,\widehat{pNua}^{Hau}_{sB,sE',sST}}wha^{Hau}_{sB,sE',sST,sRea,2}\Bigg)+\frac{\widehat{pdh}}{\hat{k}_{sST^*}\,\widehat{pNua}^{Gni}_{tran_{sB,sE',sST}}}wha^{Gni}_{sB,sE',sST,3}\\
&+\frac{\widehat{pdh}}{\hat{k}_{sST^*}\,\widehat{pNua}^{Gni}_{turb_{sB,sE',sST}}}wha^{Gni}_{sB,sE',sST,4}\Bigg]\\
&\leq\frac{\widehat{\Delta Tlm}}{\left(\frac{\widehat{Aexc}}{100}+1\right)}\sum_{sB=1}^{sBmax}\sum_{sNh=1}^{sNhmax}\sum_{sE=1}^{sEmax}\sum_{sE'=1}^{sE'max}\sum_{sST}(\widehat{pA}_{sB,sNh,,sE,sE'}\;wAF_{sB,sNh,sE,sE',sST}\\
&+\widehat{pA}_{sB,sNh,sE,sE'}\;(\widehat{pF}_{sST^*,sE}-1)wAF_{sB,sNh,sE,sE',sST}\\
&+\widehat{pA}_{sB,sNh,sE,sE'}\;(\widehat{pF}_{sST,sE'}-1)wAF_{sB,sNh,sE,sE',sST})
\end{aligned}
\tag{S162}
$$

$$wht^{theo}_{sST,sRet,sPrt,sNut}\leq yT_{sST}\tag{S163}$$

$$wht^{theo}_{sST,sRet,sPrt,sNut}\leq yRet_{sRet}\tag{S164}$$

$$wht^{theo}_{sST,sRet,sPrt,sNut}\leq yPrt_{sPrt}\tag{S165}$$

$$wht^{theo}_{sST,sRet,sPrt,sNut}\leq yNut_{sNut}\tag{S166}$$

$$wht^{theo}_{sST,sRet,sPrt,sNut}\geq yT_{sST}+yRet_{sRet}+yPrt_{sPrt}+yNut_{sNut}-3\tag{S167}$$

$$wht^{S\&T}_{sB,sE,sST,sRet,sPrt,sNut}\leq wvt_{sB,sE,sST}\tag{S168}$$

$$wht^{S\&T}_{sB,sE,sST,sRet,sPrt,sNut}\leq yRet_{sRet}\tag{S169}$$

$$wht^{S\&T}_{sB,sE,sST,sRet,sPrt,sNut}\leq yPrt_{sPrt}\tag{S170}$$

$$wht^{S\&T}_{sB,sE,sST,sRet,sPrt,sNut}\leq yNut_{sNut}\tag{S171}$$

$$
\begin{aligned}
&wht^{S\&T}_{sB,sE,sST,sRet,sPrt,sNut}\\
&\qquad\geq wvt_{sd,sB,sE,sST}+yLh_{sLh}+yRet_{sRet}+yPrt_{sPrt}+yNut_{sNut}-3
\end{aligned}
\tag{S172}
$$

$$wht^{Hau}_{sB,sE,sST,sRet,sPrt}\leq wvt_{sB,sE,sST}\tag{S173}$$

$$wht^{Hau}_{sB,sE,sST,sRet,sPrt}\leq yRet_{sRet}\tag{S174}$$

$$wht^{Hau}_{sB,sE,sST,sRet,sPrt} \leq yPrt_{sPrt} \quad (S175)$$

$$wht^{Hau}_{sB,sE,sST,sRet,sPrt} \geq wvt_{sd,sB,sE,sST} + yRet_{sRet} + yPrt_{sPrt} - 2 \quad (S176)$$

$$wht^{Gni}_{sB,sE,sST,sRet} \leq wvt_{sB,sE,sST} \quad (S177)$$

$$wht^{Gni}_{sB,sE,sST,sRet} \leq yRet_{sRet} \quad (S178)$$

$$wht^{Gni}_{sB,sE,sST,sRet} \geq wvt_{sB,sE,sST} + yRet_{sRet} - 1 \quad (S179)$$

$$wha^{theo}_{sST,sRea,sPra,sNua} \leq yT_{sST} \quad (S180)$$

$$wha^{theo}_{sST,sRea,sPra,sNua} \leq yRea_{sRea} \quad (S181)$$

$$wha^{theo}_{sST,sRea,sPra,sNua} \leq yPra_{sPra} \quad (S182)$$

$$wha^{theo}_{sST,sRea,sPra,sNua} \leq yNua_{sNua} \quad (S183)$$

$$wha^{theo}_{sST,sRea,sPra,sNua} \geq yT_{sST} + yRea_{sRea} + yPra_{sPra} + yNua_{sNua} - 3 \quad (S184)$$

$$wha^{S\&T}_{sB,sE',sST,sRea,sPra,sNua} \leq wva_{sB,sE',sST} \quad (S185)$$

$$wha^{S\&T}_{sB,sE',sST,sRea,sPra,sNua} \leq yRea_{sRea} \quad (S186)$$

$$wha^{S\&T}_{sB,sE',sST,sRea,sPra,sNua} \leq yPra_{sPra} \quad (S187)$$

$$wha^{S\&T}_{sB,sE',sST,sRea,sPra,sNua} \leq yNua_{sNua} \quad (S188)$$

$$wha^{S\&T}_{sB,sE',sST,sRea,sPra,sNua} \geq wva_{sB,sE',sST} + yRea_{sRea} + yPra_{sPra} + yNua_{sNua} - 3 \quad (S189)$$

$$wha^{Hau}_{sB,sE',sST,sRea,sPra} \leq wva_{sB,sE',sST} \quad (S190)$$

$$wha^{Hau}_{sB,sE',sST,sRea,sPra} \leq yRea_{sRea} \quad (S191)$$

$$wha^{Hau}_{sB,sE',sST,sRea,sPra} \leq yPra_{sPra} \quad (S192)$$

$$wha^{Hau}_{sB,sE',sST,sRea,sPra} \geq wva_{sB,sE',sST} + yRea_{sRea} + yPra_{sPra} - 2 \quad (S193)$$

$$wha^{Gni}_{sB,sE',sST,sRea} \leq wva_{sB,sE',sST} \quad (S194)$$

$$wha^{Gni}_{sB,sE',sST,sRea} \leq yRea_{sRea} \quad (S195)$$

$$wha^{Gni}_{sB,sE',sST,sRea} \geq wva_{sB,sE',sST} + yRea_{sRea} - 1 \quad (S196)$$

$$wAF_{sB,sNh,sE,sE\prime,sST} \leq wA_{sB,sNh,sE,sE'} \tag{S197}$$

$$wAF_{sB,sNh,sE,sE\prime,sST} \leq yT_{sST} \tag{S198}$$

$$wAF_{sB,sNh,sE,sE\prime,sST} \geq wA_{sB,sNh,sE,sE'} + yT_{sST} - 1 \tag{S199}$$

where:

$$\widehat{pNut}^{Hau}_{sB,sE,sST} = 3.66 + \frac{0{,}0668\left(\frac{8\,\widehat{Cp}_{sST}\,\widehat{m}_{sST}}{\pi\,\hat{k}_{sST}\,\widehat{pNB}_{sB}\,\widehat{pNE}_{sE}\,\widehat{pLh}}\right)}{1 + 0{,}04\left(\frac{8\,\widehat{Cp}_{sST}\,\widehat{m}_{sST}}{\pi\,\hat{k}_{sST}\,\widehat{pNB}_{sB}\,\widehat{pNE}_{sE}\,\widehat{pLh}}\right)^{2/3}} \tag{S200}$$

$$\widehat{pNut}^{Gni}_{tran_{sB,sE,sST}} = \frac{0.0061\left(\frac{4\ \widehat{m}_{sST}}{\pi\,\hat{\mu}_{sST}\,\widehat{pdt\iota}\,\widehat{pNB}_{sB}\,\widehat{pNE}_{sE}} - 1000\right)\left(\frac{\widehat{Cp}_{sST}\,\hat{\mu}_{sST}}{\hat{k}_{sST}}\right)}{1 + 12.7\left(\frac{0.0488}{8}\right)^{\frac{1}{2}}\left(\left(\frac{\widehat{Cp}_{sST}\,\hat{\mu}_{sST}}{\hat{k}_{sST}}\right)^{\frac{2}{3}} - 1\right)} \tag{S201}$$

$$\widehat{pNut}^{Gni}_{turb_{sB,sE,sST}} = \frac{\left(0.00175 + 0.132\left(\frac{\pi\,\hat{\mu}_{sST}\,\widehat{pdt\iota}\,\widehat{pNB}_{sB}\,\widehat{pNE}_{sE}}{4\ \widehat{m}_{sST}}\right)^{0.42}\right)\left(\frac{4\ \widehat{m}_{sST}}{\pi\,\hat{\mu}_{sST}\,\widehat{pdt\iota}\,\widehat{pNB}_{sB}\,\widehat{pNE}_{sE}} - 1000\right)\left(\frac{\widehat{Cp}_{sST}\,\hat{\mu}_{sST}}{\hat{k}_{sST}}\right)}{1 + 12.7\left(0.00175 + 0.132\left(\frac{\pi\,\hat{\mu}_{sST}\,\widehat{pdt\iota}\,\widehat{pNB}_{sB}\,\widehat{pNE}_{sE}}{4\ \widehat{m}_{sST}}\right)^{0.42}\right)^{1/2}\left(\left(\frac{\widehat{Cp}_{sST}\,\hat{\mu}_{sST}}{\hat{k}_{sST}}\right)^{\frac{2}{3}} - 1\right)} \tag{S202}$$

$$\widehat{pNua}^{Hau}_{sB,sE,sST} = 3.66 + \frac{0.0668\ \frac{2\,\widehat{Cp}_{sST^*}\,\widehat{m}_{sST^*}\,\widehat{pdh}^2}{\hat{k}_{sST^*}\,\widehat{pAa}\,\widehat{pNB}_{sB}\widehat{pNE}_{sE}\widehat{pLh}}}{1 + 0.04\left(\frac{2\,\widehat{Cp}_{sST^*}\,\widehat{m}_{sST^*}\,\widehat{pdh}^2}{\hat{k}_{sST^*}\,\widehat{pAa}\,\widehat{pNB}_{sB}\widehat{pNE}_{sE}\widehat{pLh}}\right)^{\frac{2}{3}}} \tag{S203}$$

$$\widehat{pNua}^{Gni}_{tran_{sB,sE,sST}} = \frac{\left(0.00337 + 4.082\left(\frac{\hat{\mu}_{sST^*}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}{\widehat{m}_{sST^*}\,\widehat{pdh}}\right)^{0.93}\right)\left(\frac{\widehat{m}_{sST^*}\,\widehat{pdh}}{\hat{\mu}_{sST^*}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}} - 1000\right)\frac{\widehat{Cp}_{sST^*}\hat{\mu}_{sST^*}}{\hat{k}_{sST^*}}}{1 + 12.7\left(0.00337 + 4.082\left(\frac{\hat{\mu}_{sST^*}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}{\widehat{m}_{sST^*}\,\widehat{pdh}}\right)^{0.93}\right)^{\frac{1}{2}}\left(\left(\frac{\widehat{Cp}_{sST^*}\hat{\mu}_{sST^*}}{\hat{k}_{sST^*}}\right)^{\frac{2}{3}} - 1\right)} \tag{S204}$$

$$\widehat{pNua}^{Gni}_{turb_{sB,sE,sST}} = \frac{\left(0.02225\left(\frac{\hat{\mu}_{sST^*}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}{\widehat{m}_{sST^*}\ \widehat{pdh}}\right)^{0.1865}\right)\left(\frac{\widehat{m}_{sST^*}\ \widehat{pdh}}{\hat{\mu}_{sST^*}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}} - 1000\right)\frac{\widehat{Cp}_{sST^*}\hat{\mu}_{sST^*}}{\hat{k}_{sST^*}}}{1 + 12.7\left(0.02225\left(\frac{\hat{\mu}_{sST^*}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}{\widehat{m}_{sST^*}\ \widehat{pdh}}\right)^{0.1865}\right)^{\frac{1}{2}}\left(\left(\frac{\widehat{Cp}_{sST^*}\hat{\mu}_{sST^*}}{\hat{k}_{sST^*}}\right)^{\frac{2}{3}} - 1\right)} \tag{S205}$$

where if $sST = h$, then $sST^* = c$ and vice-versa.

Bounds on flow velocity and pressure drop are given by:

$$vt \geq \widehat{vt}_{min} \tag{S206}$$

$$vt \leq \widehat{vt}_{max} \tag{S207}$$

$$va \geq \widehat{va}_{min} \tag{S208}$$

$$va \leq \widehat{va}_{max} \tag{S209}$$

$$\Delta Pt \leq \widehat{\Delta P}_{c_{disp}} yT_c + \widehat{\Delta P}_{h_{disp}} yT_h \tag{S210}$$

$$\Delta Pa \leq \widehat{\Delta P}_{c_{disp}} yT_h + \widehat{\Delta P}_{h_{disp}} yT_c \tag{S211}$$

The objective function is given by:

$$\min A + \widehat{ph} Nh \tag{S212}$$

Par-FLU-MILM. For the second alternative, in addition to the geometric variables (*dte, Dte* and *Lh*), the fluid allocation (yT_{sST}) is also transformed into a parameter ($\widehat{pdte}$, $\widehat{pDte}$, $\widehat{pLh}$ and $\widehat{yT_{sST}}$).

The structure selection remains the same as in Eqs. (S93) to (S97). The flow velocity inside the inner tube, its corresponding Reynolds number, the evaluation of the Nusselt number by the Sieder & Tate correlation and additional linear inequalities are:

$$vt = \sum_{sB=1}^{sBmax} \sum_{sE=1}^{sEmax} \frac{4\,\widehat{mt}}{\pi \widehat{pdti}^{\,2}\ \widehat{\rho t}\ \widehat{pNB}_{sB} \widehat{pNE}_{sE}}\ wvt_{sB,sE} \tag{S213}$$

$$Ret = \sum_{sB=1}^{sBmax} \sum_{sE=1}^{sEmax} \frac{4\,\widehat{mt}}{\pi \widehat{pdti}\ \widehat{\mu t} \widehat{pNB}_{sB} \widehat{pNE}_{sE}}\ wvt_{sB,sE} \tag{S214}$$

$$Nut^{S\&T} = \sum_{sB=1}^{sBmax} \sum_{sE=1}^{sEmax} \widehat{pNut}_{sB,sE}^{S\&T} wvt_{sB,sE} \tag{S215}$$

$$wvt_{sB,sE} \leq yB_{sB} \tag{S216}$$

$$wvt_{sB,sE} \leq yPt_{sE} \tag{S217}$$

$$wvt_{sB,sE} \geq yB_{sB} + yPt_{sE} - 1 \tag{S218}$$

where the parameter $\widehat{pNut}_{sB,sE}^{S\&T}$, is given by:

$$\widehat{pNut}^{S\&T}_{sB,sE} = 1.86\left(\frac{8\,\widehat{mt}\,\widehat{Prt}}{\pi\,\widehat{\mu t}\,\widehat{pNB}_{sB}\widehat{pNE}_{sE}\widehat{pLh}}\right)^{1/3} \tag{S219}$$

The constraints relating binary variables to Re ranges for the tube-side remain the same as in Eqs. (S112) to (S120). The pressure drop in the tubes becomes:

$$\Delta Pt = \sum_{sB=1}^{sBmax}\sum_{sNh=1}^{sNhmax}\sum_{sE=1}^{sEmax}\sum_{sE'=1}^{sEmax}\Big(\widehat{pdPt}1_{sB,sE,sNh,sE'}wdPt_{sB,sNh,sE,sE',1} + \widehat{pdPt}23_{sB,sE,sNh,sE'}wdPt_{sB,sNh,sE,sE',2} + \widehat{pdPt}23_{sB,sE,sNh,sE'}wdPt_{sB,sNh,sE,sE',3} + \widehat{pdPt}4_{sB,sE,sNh,sE'}wdPt_{sB,sNh,sE,sE',4}\Big) \tag{S220}$$

$$wdPt_{sB,sNh,sE,sE',sRet} \le wA_{sB,sNh,sE,sE'} \tag{S221}$$

$$wdPt_{sB,sNh,sE,sE',sRet} \le yRet_{sRet} \tag{S222}$$

$$wdPt_{sB,sNh,sE,sE',sRet} \ge wA_{sB,sNh,sE,sE'} + yRet_{sRet} - 1 \tag{S223}$$

The additional parameters inserted for simplification purposes in Eq. (S220) are given by:

$$\widehat{pdPt}1_{sB,sE,sNh,sE'} = \frac{128\,\widehat{\mu t}\,\widehat{mt}\widehat{pNh}_{sNh}\widehat{pLh}\,\widehat{pNE}_{sE'}}{\pi\,\widehat{\rho t}\,\widehat{pdti}^4\widehat{pNB}_{sB}\widehat{pNE}_{sE}} \tag{S224}$$

$$\widehat{pdPt}23_{sB,sE,sNh,sE'} = \frac{0.3904\,\widehat{mt}^2\widehat{pNh}_{sNh}\widehat{pLh}\,\widehat{pNE}_{sE'}}{\pi^2\,\widehat{\rho t}\,\widehat{pdti}^5\widehat{pNB}^2_{sB}\widehat{pNE}^2_{sE}} \tag{S225}$$

$$\widehat{pdPt}4_{sB,sE,sNh,sE'} = \frac{0.112\,\widehat{mt}^2\widehat{pNh}_{sNh}\widehat{pLh}\,\widehat{pNE}_{sE'}}{\pi^2\,\widehat{\rho t}\,\widehat{pdti}^5\widehat{pNB}^2_{sB}\widehat{pNE}^2_{sE}} + \frac{4.719\,\widehat{\mu t}^{0.42}\,\widehat{mt}^{1.58}\widehat{pNh}_{sNh}\widehat{pLh}\widehat{pNE}_{sE'}}{\pi^{1.58}\,\widehat{\rho t}\,\widehat{pdti}^{4.58}\widehat{pNB}^{1.58}_{sB}\widehat{pNE}^{1.58}_{sE}} \tag{S226}$$

The annulus-side stream velocity, its corresponding Reynolds number, the Nusselt number evaluation by the Sieder & Tate correlation and additional linear inequalities are given by:

$$va = \sum_{sB=1}^{sBmax}\sum_{sE=1}^{sEmax}\frac{\widehat{ma}}{\widehat{\rho a}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}wva_{sB,sE} \tag{S227}$$

$$Rea = \sum_{sB=1}^{sBmax} \sum_{sE=1}^{sEmax} \frac{\widehat{pdh}\,\widehat{ma}}{\widehat{\mu a}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}} wva_{sB,sE} \tag{S228}$$

$$Nua^{S\&T} = \sum_{sB=1}^{sBmax} \sum_{sE=1}^{sEmax} \widehat{pNua}^{S\&T}_{sB,sE} wva_{sB,sE} \tag{S229}$$

$$wva_{sB,sE} \leq yB_{sB} \tag{S230}$$

$$wva_{sB,sE} \leq yPa_{sE} \tag{S231}$$

$$wva_{sB,sE} \geq yB_{sB} + yPa_{sE} - 1 \tag{S232}$$

where the parameter $\sum_{sST} \widehat{pNua}^{S\&T}_{sB,sE}$ is given by:

$$\widehat{pNua}^{S\&T}_{sB,sE} = 1.86 \left(\frac{2\,\widehat{pdh}^2\,\widehat{ma}\,\widehat{Pra}}{\widehat{\mu a}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}\widehat{pLh}} \right)^{\frac{1}{3}} \tag{S233}$$

The constraints relating binary variables to Re ranges for the annular-side remain the same as in Eqs. (S138) to (S146). The pressure drop of the flow in the annulus is given by:

$$\begin{aligned} \Delta Pa = \sum_{sB=1}^{sBmax} \sum_{sE=1}^{sEmax} \sum_{sNh=1}^{sNhmax} \sum_{sE'=1}^{sEmax} & (\widehat{pdPa}1_{sB,sE,sNh,sE'} wdPa_{sB,sNh,sE.sE',1} \\ & + \widehat{pdPa}23_{sB,sE,sNh,sE'} (wdPa_{sB,sNh,sE.sE',sRea} \\ & + wdPa_{sB,sNh,sE.sE',3}) + \widehat{pdPa}4_{sB,sE,sNh,sE'} wdPa_{sB,sNh,sE.sE',4} \end{aligned} \tag{S234}$$

$$wdPa_{sB,sNh,sE.sE',sRea} \leq wA_{sB,sNh,sE,sE'} \tag{S235}$$

$$wdPa_{sB,sNh,sE.sE',sRea} \leq yRea_{sRea} \tag{S236}$$

$$wdPa_{sB,sNh,sE.sE',sRea} \geq wA_{sB,sNh,sE,sE'} + yRea_{sRea} - 1 \tag{S237}$$

$$\widehat{pdPa}1_{sB,sE,sNh,sE'} = \frac{32\,\widehat{\mu a}\,\widehat{ma}\,\widehat{pNh}_{sNh}\widehat{pLh}\,\widehat{pNE}_{sE}}{\widehat{\rho a}\,\widehat{pdh^2}\,\widehat{pAa}\,\widehat{pNB}\,\widehat{pNE}_{sE'}} \tag{S238}$$

$$\widehat{pdPa}23_{sB,sE,sNh,sE'} = \frac{0.01348\,\widehat{ma}^2\,\widehat{pNh}_{sNh}\widehat{pLhpNE}_{sE}}{\widehat{\rho a}\,\widehat{pdh}\,\widehat{pAa}^2\widehat{pNB}_{sB}^2\widehat{pNE}_{sE'}^2} + \frac{16.328\widehat{\mu a}^{0.93}\,\widehat{ma}^{1.07}\,\widehat{pNh}_{sNh}\widehat{pLhpNE}_{sE}}{\widehat{\rho a}\,\widehat{pdh}^{1.93}\,\widehat{pAa}^{1.07}\widehat{pNB}_{sB}^{1.07}\widehat{pNE}_{sE'}^{1.07}} \quad \text{(S239)}$$

$$\widehat{pdPa}4_{sB,sE,sNh,sE'} = \frac{0.089\,\widehat{\mu a}^{0.1865}\,\widehat{ma}^{1.8135}\,\widehat{pNh}_{sNh}\widehat{pLh}_{sLh}\widehat{pNE}_{sE}}{\widehat{\rho a}\,\widehat{pdh}^{1.1865}\,\widehat{pAa}^{1.8135}\widehat{pNB}_{sB}^{1.8135}\widehat{pNE}_{sE'}^{1.8135}} \quad \text{(S240)}$$

The equipment heat transfer area is still given by Eqs. (S155) to (S161). The heat transfer rate, based on the LMTD method, after reformulation becomes:

$$\hat{Q} \sum_{sB=1}^{sBmax}\sum_{sE=1}^{sEmax}\sum_{sE'=1}^{sEmax}\left[\sum_{sRet=1}^{2}\left(\frac{\widehat{pdte}\,\widehat{pyPrt}_1}{\widehat{ktpNut}^{theo}} wht_{sRet,sNut}^{theo} + \frac{\widehat{pdte}\,\widehat{pyPrt}_1}{\widehat{ktpNut}_{sB,sE}^{S\&T}} wht_{sB,sE,sRet,sNut}^{S\&T} + \frac{\widehat{pdte}\,\widehat{pyPrt}_2}{\widehat{ktpNut}_{sB,sE}^{Hau}} wht_{sB,sE,sRet}^{Hau}\right) + \frac{\widehat{pdte}}{\widehat{ktpNut}_{tran_{sB,sE}}^{Gni}}\, wht_{sB,sE,sRet}^{Gni} + \frac{\widehat{pdte}}{\widehat{ktpNut}_{turb_{sB,sE}}^{Gni}}\, wht_{sB,sE,sRet}^{Gni} + \widehat{Rft}\frac{\widehat{pdte}}{\widehat{pdti}} + \frac{\widehat{pdte}\ln\left(\frac{\widehat{pdte}}{\widehat{pdti}}\right)}{2\widehat{ktube}} + \widehat{Rfa} + \sum_{sRea=1}^{2}\left(\frac{\widehat{pdh}\,\widehat{pyPra}_1}{\widehat{kapNua}^{theo}} wha_{sRea,sNua}^{theo} + \frac{\widehat{pdh}\,\widehat{pyPra}_1}{\widehat{kapNua}_{sB,sE}^{S\&T}} wha_{sB,sE',sRea,sNua}^{S\&T} + \frac{\widehat{pdh}\,\widehat{pyPra}_2}{\widehat{kapNua}_{sB,sE}^{Hau}} wha_{sB,sE',sRea}^{Hau}\right) + \frac{\widehat{pdh}}{\widehat{kapNua}_{tran_{sB,sE'}}^{Gni}} wha_{sB,sE',sRea}^{Gni} + \frac{\widehat{pdh}}{\widehat{kapNua}_{turb_{sB,sE'}}^{Gni}} wha_{sB,sE',sRea}^{Gni}\right] \le \frac{\widehat{\Delta Tlm}}{\left(\frac{\widehat{Aexc}}{100}+1\right)} \sum_{sB=1}^{sBmax}\sum_{sNh=1}^{sNhmax}\sum_{sE=1}^{sEmax}\sum_{sE'=1}^{sE'max} \widehat{pA}_{sB,sNh,sE,sE'}\, wA_{sB,sNh,sE,sE'} + \widehat{pA}_{sB,sNh,sE,sE'}\,\left(\widehat{pFa}_{sE}-1\right) wA_{sB,sNh,sE,sE'} + \widehat{pA}_{sB,sNh,sE,sE'}\,\left(\widehat{pFt}_{sE'}-1\right) wA_{sB,sNh,sE,sE'}) \quad \text{(S241)}$$

$$wht_{sRet,sNut}^{theo} \le yRet_{sRet} \quad \text{(S242)}$$

$$wht_{sRet,sNut}^{theo} \le yNut_{sNut} \quad \text{(S243)}$$

$$wht_{sRet,sNut}^{theo} \ge yRet_{sRet} + yNut_{sNut} - 1 \quad \text{(S244)}$$

$$wht_{sB,sE,sRet,sNut}^{S\&T} \le wht_{sB,sE,sRet}^{Hau} \quad \text{(S245)}$$

$$wht^{S\&T}_{sB,sE,sRet,sNut} \leq yNut_{sNut} \tag{S246}$$

$$wht^{S\&T}_{sB,sE,sRet,sNut} \geq wht^{Hau}_{sB,sE,sRet} + yNut_{sNut} - 1 \tag{S247}$$

$$wht^{Hau}_{sB,sE,sRet} \leq wvt_{sB,sE} \tag{S248}$$

$$wht^{Hau}_{sB,sE,sRet} \leq yRet_{sRet} \tag{S249}$$

$$wht^{Hau}_{sB,sE,sRet} \geq wvt_{sd,sB,sE} + yRet_{sRet} - 1 \tag{S250}$$

$$wht^{Gni}_{sB,sE,sRet} \leq wvt_{sB,sE} \tag{S251}$$

$$wht^{Gni}_{sB,sE,sRet} \leq yRet_{sRet} \tag{S252}$$

$$wht^{Gni}_{sB,sE,sRet} \geq wvt_{sB,sE} + yRet_{sRet} - 1 \tag{S253}$$

$$wha^{theo}_{sRea,sNua} \leq yRea_{sRea} \tag{S254}$$

$$wha^{theo}_{sRea,sNua} \leq yNua_{sNua} \tag{S255}$$

$$wha^{theo}_{sRea,sNua} \geq yRea_{sRea} + yNua_{sNua} - 1 \tag{S256}$$

$$wha^{S\&T}_{sB,sE',sRea,sNua} \leq wha^{Hau}_{sB,sE',sRea} \tag{S257}$$

$$wha^{S\&T}_{sB,sE',sRea,sNua} \leq yNua_{sNua} \tag{S258}$$

$$wha^{S\&T}_{sB,sE',sRea,sNua} \geq wha^{Hau}_{sB,sE',sRea} + yNua_{sNua} - 1 \tag{S259}$$

$$wha^{Hau}_{sB,sE',sRea} \leq wva_{sB,sE'} \tag{S260}$$

$$wha^{Hau}_{sB,sE',sRea} \leq yRea_{sRea} \tag{S261}$$

$$wha^{Hau}_{sB,sE',sRea} \geq wva_{sB,sE'} + yRea_{sRea} - 1 \tag{S262}$$

$$wha^{Gni}_{sB,sE',sRea} \leq wva_{sB,sE'} \tag{S263}$$

$$wha^{Gni}_{sB,sE',sRea} \leq yRea_{sRea} \tag{S264}$$

$$wha^{Gni}_{sB,sE',sRea} \geq wva_{sB,sE'} + yRea_{sRea} - 1 \tag{S265}$$

where:

$$\widehat{pNut}_{sB,sE}^{Hau} = 3.66 + \frac{0{,}0668\left(\frac{8\,\widehat{mt}\,\widehat{Prt}}{\pi\,\widehat{\mu t}\widehat{pNB}_{sB}\widehat{pNE}_{sE}\widehat{pLh}}\right)}{1 + 0{,}04\left(\frac{8\,\widehat{mt}\,\widehat{Prt}}{\pi\,\widehat{\mu t}\widehat{pNB}_{sB}\widehat{pNE}_{sE}\widehat{pLh}}\right)^{2/3}} \tag{S266}$$

$$\widehat{pNut}_{tran_{sB,sE}}^{Gni} = \frac{0.0061\left(\frac{4\,\widehat{mt}}{\pi\widehat{pdti}\,\widehat{\mu t}\widehat{pNB}_{sB}\widehat{pNE}_{sE}} - 1000\right)\widehat{Prt}}{1 + 12.7\left(\frac{0.0488}{8}\right)^{\frac{1}{2}}\left(\widehat{Prt}^{\frac{2}{3}} - 1\right)} \tag{S267}$$

$$\widehat{pNut}_{turb_{sB,sE}}^{Gni} = \frac{\left(0.00175 + 0.132\left(\frac{\pi\,\widehat{\mu t}\,\widehat{pdti}\,\widehat{pNB}_{sB}\,\widehat{pNE}_{sE}}{4\,\widehat{mt}}\right)^{0.42}\right)\left(\frac{4\,\widehat{mt}}{\pi\widehat{pdti}\,\widehat{\mu t}\,\widehat{pNB}_{sB}\widehat{pNE}_{sE}} - 1000\right)\widehat{Prt}}{1 + 12.7\left(0.00175 + 0.132\left(\frac{\pi\,\widehat{\mu t}\,\widehat{pdti}\,\widehat{pNB}_{sB}\,\widehat{pNE}_{sE}}{4\,\widehat{mt}}\right)^{0.42}\right)^{1/2}\left(\widehat{Prt}^{\frac{2}{3}} - 1\right)} \tag{S268}$$

$$\widehat{pNua}_{sB,sE}^{Hau} = 3.66 + \frac{0.0668\,\frac{2\,\widehat{pdh}^{2}\,\widehat{ma}\,\widehat{Pra}}{\widehat{\mu a}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}\widehat{pLh}}}{1 + 0.04\left(\frac{2\,\widehat{pdh}^{2}\,\widehat{ma}\,\widehat{Pra}}{\widehat{\mu a}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}\widehat{pLh}}\right)^{\frac{2}{3}}} \tag{S269}$$

$$\widehat{pNua}_{tran_{sB,sE}}^{Gni} = \frac{\left(0.00337 + 4.082\left(\frac{\widehat{\mu a}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}{\widehat{pdh}\,\widehat{ma}}\right)^{0.93}\right)\left(\frac{\widehat{pdh}\,\widehat{ma}}{\widehat{\mu a}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}} - 1000\right)\widehat{Pra}}{1 + 12.7\left(0.00337 + 4.082\left(\frac{\widehat{\mu a}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}{\widehat{pdh}\,\widehat{ma}}\right)^{0.93}\right)^{\frac{1}{2}}\left(\widehat{Pra}^{\frac{2}{3}} - 1\right)} \tag{S270}$$

$$\widehat{pNua}_{turb_{sB,sE}}^{Gni} = \frac{\left(0.02225\left(\frac{\widehat{\mu a}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}{\widehat{pdh}\,\widehat{ma}}\right)^{0.1865}\right)\left(\frac{\widehat{pdh}\,\widehat{ma}}{\widehat{\mu a}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}} - 1000\right)\widehat{Pra}}{1 + 12.7\left(0.02225\left(\frac{\widehat{\mu a}\,\widehat{pAa}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}{\widehat{pdh}\,\widehat{ma}}\right)^{0.1865}\right)^{\frac{1}{2}}\left(\widehat{Pra}^{\frac{2}{3}} - 1\right)} \tag{S271}$$

Bounds on flow velocity are still given by Eqs. (S206) to (S211). Pressure drop bounds are given by:

$$\Delta Pt \leq \Delta\widehat{Pt_{disp}} \tag{S272}$$

$$\Delta Pa \leq \Delta\widehat{Pa_{disp}} \tag{S273}$$

The objective function is the same as in Eq. (S212).

Par-STR-MILM. For the third alternative, the structural variables (*NB, NPt*, *NPa* and *Nh*) are transformed into parameters ($\widehat{pNB}$, $\widehat{pNPt}$, $\widehat{pNPa}$ and $\widehat{pNh}$).

The hairpin dimensions are selected via the following constraints:

$$\sum_{sd=1}^{sdmax} yd_{sd} = 1 \tag{S274}$$

$$\sum_{sD=1}^{sDmax} yD_{sD} = 1 \tag{S275}$$

$$\sum_{sLh=1}^{sLhmax} yLh_{sLh} = 1 \tag{S276}$$

$$yd_{sd} + yD_{sD} \leq 1 \tag{S277}$$

The stream allocation is controlled by the binary variables yT_c and yT_h:

$$yT_c + yT_h = 1 \tag{S278}$$

The flow velocity inside the inner tube, its corresponding Reynolds number and additional linear inequalities are:

$$vt = \sum_{sd=1}^{sdmax} \sum_{sST} \frac{4\,\widehat{m}_{sST}}{\pi\,\widehat{\rho}_{sST}\,\widehat{pdti}_{sd}^{2}\widehat{pNB}\widehat{pNPt}}\, wydT_{sd,sST} \tag{S279}$$

$$Ret = \sum_{sd=1}^{sdmax} \sum_{sST} \frac{4\,\widehat{m}_{sST}}{\pi\,\widehat{\mu}_{sST}\,\widehat{pdti}_{sd}\widehat{pNB}\widehat{pNPt}}\, wydT_{sd,sST} \tag{S280}$$

$$wydT_{sd,sST} \leq yd_{sd} \tag{S281}$$

$$wydT_{sd,sST} \leq yT_{sST} \tag{S282}$$

$$wydT_{sd,sST} \geq yd_{sd} + yT_{sST} - 1 \tag{S283}$$

The Prandtl number of the inner tube stream is the same as in Eq. (S105). The evaluation of the Nusselt number by the Sieder & Tate correlation is now:

$$Nut^{S\&T} = \sum_{sLh=1}^{sLhmax} \sum_{sST} \widehat{pNut}_{sLh,sST}^{S\&T} wNut_{sLh,sST} \tag{S284}$$

$$wNut_{sLh,sST} \leq yLh_{sLh} \tag{S285}$$

$$wNut_{sLh,sST} \leq yT_{sST} \tag{S286}$$

$$wNut_{sLh,sST} \geq yLh_{sLh} + yT_{sST} - 1 \tag{S287}$$

$$\widehat{pNut}^{S\&T}_{sLh,sST} = 1.86\left(\frac{8\,\widehat{Cp}_{sST}\,\widehat{m}_{sST}}{\pi\,\hat{k}_{sST}\,\widehat{pNB}\widehat{pNPt}\,\widehat{pLh}_{sLh}}\right)^{1/3} \tag{S288}$$

The constraints relating binary variables to Re ranges for the tube-side remain the same as Eqs. (S112) to (S120). The pressure drop in the tubes becomes:

$$\Delta Pt = \sum_{sd=1}^{sdmax}\sum_{sLh=1}^{sLhmax}\sum_{sST}\Big(\widehat{pdPt}1_{sd,sLh,sST}wdPt_{sd,sLh,sST,1} + \widehat{pdPt}23_{sd,sLh,sST}wdPt_{sd,sLh,sST,2} + \widehat{pdPt}23_{sd,sLh,sST}wdPt_{sd,sLh,sST,3} + \widehat{pdPt}4_{sd,sLh,sST}wdPt_{sd,sLh,sST,4}\Big) \tag{S289}$$

$$wdPt_{sd,sLh,sST,sRet} \leq yT_{sST} \tag{S290}$$

$$wdPt_{sd,sLh,sST,sRet} \leq yRet_{sRet} \tag{S291}$$

$$wdPt_{sd,sLh,sST,sRet} \geq wA_{sd,sLh} + yT_{sST} + yRet_{sRet} - 2 \tag{S292}$$

$$\widehat{pdPt}1_{sd,sLh,sST} = \frac{128\,\hat{\mu}_{sST}\,\widehat{m}_{sST}\widehat{pNh}\,\widehat{pLh}_{sLh}\widehat{pNPa}}{\pi\,\hat{\rho}_{sST}\,\widehat{pdti}^{4}_{sd}\widehat{pNB}\widehat{pNPt}} \tag{S293}$$

$$\widehat{pdPt}23_{sd,sLh,sST} = \frac{0.3904\,\widehat{m}^{2}_{sST}\widehat{pNh}\,\widehat{pLh}_{sLh}\widehat{pNPa}}{\pi^{2}\,\hat{\rho}_{sST}\,\widehat{pdti}^{5}_{sd}\widehat{pNB}^{2}\widehat{pNPt}^{2}} \tag{S294}$$

$$\widehat{pdPt}4_{sd,sLh,sST} = \frac{0.112\,\widehat{m}^{2}_{sST}\widehat{pNh}\,\widehat{pLh}_{sLh}\widehat{pNPa}}{\pi^{2}\,\hat{\rho}_{sST}\,\widehat{pdti}^{5}_{sd}\widehat{pNB}^{2}\widehat{pNPt}^{2}} + \frac{4.719\,\hat{\mu}^{0.42}_{sST}\,\widehat{m}^{1.58}_{sST}\widehat{pNh}\,\widehat{pLh}_{sLh}\widehat{pNPa}}{\pi^{1.58}\,\hat{\rho}_{sST}\,\widehat{pdti}^{4.58}_{sd}\widehat{pNB}^{1.58}\widehat{pNPt}^{1.58}} \tag{S295}$$

The annulus-side stream velocity, its corresponding Reynolds number and additional linear inequalities are given by:

$$va = \sum_{sd=1}^{sdmax}\sum_{sD=1}^{sDmax}\sum_{sST}\frac{\widehat{m}_{sST^*}}{\hat{\rho}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}\,\widehat{pNPa}}wva_{sd,sD,sST} \tag{S296}$$

$$Rea = \sum_{sd=1}^{sdmax}\sum_{sD=1}^{sDmax}\sum_{sST}\frac{\widehat{m}_{sST^*}\,\widehat{pdh}_{sd,sD}}{\hat{\mu}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}\,\widehat{pNPa}}wva_{sd,sD,sST} \tag{S297}$$

$$wva_{sd,sD,sST} \leq wyTd_{sd,sST} \quad \text{(S298)}$$

$$wva_{sd,sD,sST} \leq yD_{sD} \quad \text{(S299)}$$

$$wva_{sd,sD,sST} \geq wyTd_{sd,sST} + yT_{sST} - 1 \quad \text{(S300)}$$

where if $sST = h$, then $sST^{*} = c$ and vice-versa.

The Prandtl number for the annulus-side stream becomes is the same as in Eq (S135). The Nusselt number evaluation by the Sieder & Tate correlation becomes:

$$Nua^{S\&T} = \sum_{sd=1}^{sdmax} \sum_{sD=1}^{sDmax} \sum_{sLh=1}^{sLhmax} \sum_{sST} \widehat{pNua}_{sd,sD,sLh,sST}^{S\&T} \, wNua_{sd,sD,sLh.sST} \quad \text{(S301)}$$

$$wNua_{sd,sD,sLh.sST} \leq wva_{sd,sD,sST} \quad \text{(S302)}$$

$$wNua_{sd,sD,sLh.sST} \leq yLh_{sLh} \quad \text{(S303)}$$

$$wNua_{sd,sD,sLh.sST} \geq wva_{sd,sD,sST} + yLh_{sLh} - 1 \quad \text{(S304)}$$

$$\widehat{pNua}_{sd,sD,sLh,sST}^{S\&T} = 1.86 \left(\frac{2\, \widehat{Cp}_{sST^{*}} \, \widehat{m}_{sST^{*}} \, \widehat{pdh}_{sd,sD}^{2}}{\widehat{k}_{sST^{*}} \, \widehat{pAa}_{sd,sD} \widehat{pNB} \, \widehat{pNPa} \, \widehat{pLh}_{sLh}} \right)^{\frac{1}{3}} \quad \text{(S305)}$$

where if $sST = h$, then $sST^{*} = c$ and vice-versa.

The constraints relating binary variables to Re ranges for the annular-side remain the same as in Eqs. (S138) to (S146). The pressure drop of the flow in the annulus is given by:

$$\begin{aligned} \Delta Pa = \sum_{sd=1}^{sdmax} \sum_{sD=1}^{sDmax} \sum_{sLh=1}^{sLhmax} \sum_{sST} (&\widehat{pdPa}1_{sd,sD,sLh,sST} wdPa_{sd,sD,sLh,sST,1} \\ &+ \widehat{pdPa}23_{sd,sD,sLh,sST} wdPa_{sd,sD,sLh,sST,2} \\ &+ \widehat{pdPa}23_{sd,sD,sLh,sST} wdPa_{sd,sD,sLh,sST,3} \\ &+ \widehat{pdPa}4_{sd,sD,sLh,sST} wdPa_{sd,sD,sLh,sST,4}) \end{aligned} \quad \text{(S306)}$$

$$wdPa_{sd,sD,sLh,sST,sRea} \leq wA_{sd,sLh} \quad \text{(S307)}$$

$$wdPa_{sd,sD,sLh,sST,sRea} \leq yD_{sD} \quad \text{(S308)}$$

$$wdPa_{sd,sD,sLh,sST,sRea} \leq yT_{sST} \quad \text{(S309)}$$

$$wdPa_{sd,sD,sLh,sST,sRea} \leq yRea_{sRea} \tag{S310}$$

$$wdPa_{sd,sD,sLh,sST,sRea} \geq wA_{sd,sLh} + yD_{sD} + yT_{sST} + yRea_{sRea} - 3 \tag{S311}$$

$$\widehat{pdPa}1_{sd,sD,sLh,sST} = \frac{32\ \hat{\mu}_{sST^*}\ \widehat{m}_{sST^*}\ \widehat{pLh}_{sLh}\widehat{pNh}\ \widehat{pNPt}}{\hat{\rho}_{sST^*}\ \widehat{pdh}^2_{sd,sD}\ \widehat{pAa}_{sd,sD}\widehat{pNB}\ \widehat{pNPa}} \tag{S312}$$

$$\widehat{pdPa}23_{sd,sD,sLh,sST} = \frac{0.01348\ \widehat{m}^2_{sST^*}\ \widehat{pLh}_{sLh}\widehat{pNh}\ \widehat{pNPt}}{\hat{\rho}_{sST^*}\ \widehat{pdh}_{sd,sD}\ \widehat{pAa}^2_{sd,sD}\widehat{pNB}^2\ \widehat{pNPa}^2} + \frac{16.328\ \hat{\mu}^{0.93}_{sST^*}\ \widehat{m}^{1.07}_{sST^*}\ \widehat{pLh}_{sLh}\widehat{pNh}\ \widehat{pNPt}}{\hat{\rho}_{sST^*}\ \widehat{pdh}^{1.93}_{sd,sD}\ \widehat{pAa}^{1.07}_{sd,sD}\widehat{pNB}^{1.07}\ \widehat{pNPa}^{1.07}} \tag{S313}$$

$$\widehat{pdPa}4_{sd,sD,sLh,sST} = \frac{0.089\ \hat{\mu}^{0.1865}_{sST^*}\ \widehat{m}^{1.8135}_{sST^*}\ \widehat{pLh}_{sLh}\widehat{pNh}\ \widehat{pNPt}}{\hat{\rho}_{sST^*}\ \widehat{pdh}^{1.1865}_{sd,sD}\ \widehat{pAa}^{1.8135}_{sd,sD}\widehat{pNB}^{1.8135}\ \widehat{pNPa}^{1.8135}} \tag{S314}$$

where if $sST = h$, then $sST^* = c$ and vice-versa.

The equipment heat transfer area is given by:

$$A = \sum_{sd=1}^{sdmax} \sum_{sLh=1}^{sLhmax} \widehat{pA}_{sd,sLh}\ wA_{sd,sLh} \tag{S315}$$

$$wA_{sd,sLh} \leq yd_{sd} \tag{S316}$$

$$wA_{sd,sLh} \leq yLh_{sLh} \tag{S317}$$

$$wA_{sd,sLh} \geq yd_{sd} + yLh_{sLh} - 1 \tag{S318}$$

$$\widehat{pA}_{sd,sLh} = \pi\ \widehat{pdte}_{sd}\ \widehat{pNB}\ \widehat{pNh}\ \widehat{pLh}_{sLh}\widehat{pNPt}\ \widehat{pNPa} \tag{S319}$$

The heat transfer rate, based on the LMTD method, after reformulation becomes:

$$\hat{Q}\sum_{sd=1}^{sdmax}\sum_{sD=1}^{sDmax}\sum_{sLh=1}^{sLhmax}\sum_{sST}\Bigg[\sum_{sRet=1}^{2}\Bigg(\frac{\widehat{pdte}_{sd}}{\hat{k}_{sST}\,\widehat{pNut}^{theo}}wht^{theo}_{sd,sST,sRet,1,1} + \frac{\widehat{pdte}_{sd}}{\hat{k}_{sST}\,\widehat{pNut}^{S\&T}_{sLh,sST}}wht^{S\&T}_{sd,sLh,sST,sRet,1,2} + \frac{\widehat{pdte}_{sd}}{\hat{k}_{sST}\,\widehat{pNut}^{Hau}_{sLh,sST}}wht^{Hau}_{sd,sLh,sST,sRet,2}\Bigg) + \frac{\widehat{pdte}_{sd}}{\hat{k}_{sST}\,\widehat{pNut}^{Gni}_{tran_{sd,sST}}}wht^{Gni}_{sd,sST,3} + \frac{\widehat{pdte}_{sd}}{\hat{k}_{sST}\,\widehat{pNut}^{Gni}_{turb_{sd,sST}}}wht^{Gni}_{sd,sST,4} + \widehat{Rf}_{sST}\frac{\widehat{pdte}_{sd}}{\widehat{pdti}_{sd}}wydT_{sd,sST} + \frac{\widehat{pdte}_{sd}\ln\left(\frac{\widehat{pdte}_{sd}}{\widehat{pdti}_{sd}}\right)}{2\widehat{ktube}}yd_{sd} + \widehat{Rf}_{sST^*}yT_{sST} + \sum_{sRea=1}^{2}\Bigg(\frac{\widehat{pdh}_{sd,sD}}{\hat{k}_{sST^*}\,\widehat{pNua}^{theo}}wha^{theo}_{sd,sD,sST,sRea,1,1} + \frac{\widehat{pdh}_{sd,sD}}{\hat{k}_{sST^*}\,\widehat{pNua}^{S\&T}_{sd,sD,sLh,sST}}wha^{theo}_{sd,sD,sST,sRea,1,2} + \frac{\widehat{pdh}_{sd,sD}}{\hat{k}_{sST^*}\,\widehat{pNua}^{Hau}_{sd,sD,sLh,sST}}wha^{Hau}_{sd,sD,sLh,sST,sRea,2}\Bigg) + \frac{\widehat{pdh}_{sd,sD}}{\hat{k}_{sST^*}\,\widehat{pNua}^{Gni}_{tran_{sd,sD,sST}}}wha^{Gni}_{sd,sD,sST,3} + \frac{\widehat{pdh}_{sd,sD}}{\hat{k}_{sST^*}\,\widehat{pNua}^{Gni}_{turb_{sd,sD,sST}}}wha^{Gni}_{sd,sD,sST,4}\Bigg] \le \frac{\widehat{\Delta Tlm}}{\left(\frac{\widehat{Aexc}}{100}+1\right)}\sum_{sd=1}^{sdmax}\sum_{sLh=1}^{sLhmax}\sum_{sST}\Big(\widehat{pA}_{sd,sLh}\,wA_{sd,sLh} + \widehat{pA}_{sd,sLh}\,\left(\widehat{pFNPt}_{sST^*}-1\right)wAF_{sd,sLh,sST} \ge +\widehat{pA}_{sd,sLh}\,\left(\widehat{pFNPa}_{sST}-1\right)wAF_{sd,sLh,sST} \ge\Big) \quad \text{(S320)}$$

$$wydT_{sd,sST} \le yd_{sd} \quad \text{(S321)}$$

$$wydT_{sd,sST} \le yT_{sST} \quad \text{(S322)}$$

$$wydT_{sd,sST} \ge yd_{sd} + yT_{sST} - 1 \quad \text{(S323)}$$

$$wht^{theo}_{sd,sST,sRet,sPrt,sNut} \le wydT_{sd,sST} \quad \text{(S324)}$$

$$wht^{theo}_{sd,sST,sRet,sPrt,sNut} \le yRet_{sRet} \quad \text{(S325)}$$

$$wht^{theo}_{sd,sST,sRet,sPrt,sNut} \le yPrt_{sPrt} \quad \text{(S326)}$$

$$wht^{theo}_{sd,sST,sRet,sPrt,sNut} \le yNut_{sNut} \quad \text{(S327)}$$

$$wht^{theo}_{sd,sST,sRet,sPrt,sNut} \ge wydT_{sd,sST} + yRet_{sRet} + yPrt_{sPrt} + yNut_{sNut} - 3 \quad \text{(S328)}$$

$$wht^{S\&T}_{sd,sLh,sST,sRet,sPrt,sNut} \le wvt_{sd,sST} \quad \text{(S329)}$$

$$wht^{S\&T}_{sd,sLh,sST,sRet,sPrt,sNut} \le yLh_{sLh} \quad \text{(S330)}$$

$$wht^{S\&T}_{sd,sLh,sST,sRet,sPrt,sNut} \le yRet_{sRet} \quad \text{(S331)}$$

$$wht^{S\&T}_{sd,sLh,sST,sRet,sPrt,sNut} \leq yPrt_{sPrt} \tag{S332}$$

$$wht^{S\&T}_{sd,sLh,sST,sRet,sPrt,sNut} \leq yNut_{sNut} \tag{S333}$$

$$wht^{S\&T}_{sd,sLh,sST,sRet,sPrt,sNut} \geq wvt_{sd,sB,sE,sST} + yLh_{sLh} + yRet_{sRet} + yPrt_{sPrt} + yNut_{sNut} - 4 \tag{S334}$$

$$wht^{Hau}_{sd,sLh,sST,sRet,sPrt} \leq wvt_{sd,sST} \tag{S335}$$

$$wht^{Hau}_{sd,sLh,sST,sRet,sPrt} \leq yLh_{sLh} \tag{S336}$$

$$wht^{Hau}_{sd,sLh,sST,sRet,sPrt} \leq yRet_{sRet} \tag{S337}$$

$$wht^{Hau}_{sd,sLh,sST,sRet,sPrt} \leq yPrt_{sPrt} \tag{S338}$$

$$wht^{Hau}_{sd,sLh,sST,sRet,sPrt} \geq wvt_{sd,sST} + yLh_{sLh} + yRet_{sRet} + yPrt_{sPrt} - 3 \tag{S339}$$

$$wht^{Gni}_{sd,sST,sRet} \leq wvt_{sd,sST} \tag{S340}$$

$$wht^{Gni}_{sd,sST,sRet} \leq yRet_{sRet} \tag{S341}$$

$$wht^{Gni}_{sd,sST,sRet} \geq wvt_{sd,sST} + yRet_{sRet} - 1 \tag{S342}$$

$$wha^{theo}_{sd,sD,sST,sRea,sPra,sNua} \leq wydT_{sd,sST} \tag{S343}$$

$$wha^{theo}_{sd,sD,sST,sRea,sPra,sNua} \leq yD_{sD} \tag{S344}$$

$$wha^{theo}_{sd,sD,sST,sRea,sPra,sNua} \leq yRea_{sRea} \tag{S345}$$

$$wha^{theo}_{sd,sD,sST,sRea,sPra,sNua} \leq yPra_{sPra} \tag{S346}$$

$$wha^{theo}_{sd,sD,sST,sRea,sPra,sNua} \leq yNua_{sNua} \tag{S347}$$

$$wha^{theo}_{sd,sD,sST,sRea,sPra,sNua} \geq wydT_{sd,sST} + yD_{sD} + yRea_{sRea} + yPra_{sPra} + yNua_{sNua} - 4 \tag{S348}$$

$$wha^{S\&T}_{sd,sD,sLh,sST,sRea,sPra,sNua} \leq wva_{sd,sD,sST} \tag{S349}$$

$$wha^{S\&T}_{sd,sD,sLh,sST,sRea,sPra,sNua} \leq yLh_{sLh} \tag{S350}$$

$$wha^{S\&T}_{sd,sD,sLh,sST,sRea,sPra,sNua} \leq yRea_{sRea} \tag{S351}$$

$$wha^{S\&T}_{sd,sD,sLh,sST,sRea,sPra,sNua} \leq yPra_{sPra} \tag{S352}$$

$$wha^{S\&T}_{sd,sD,sLh,sST,sRea,sPra,sNua} \leq yNua_{sNua} \tag{S353}$$

$$wha^{S\&T}_{sd,sD,sLh,sST,sRea,sPra,sNua} \geq wva_{sd,sD,sST} + yLh_{sLh} + yRea_{sRea} + yPra_{sPra} + yNua_{sNua} - 4 \quad \text{(S354)}$$

$$wha^{Hau}_{sd,sD,sLh,sST,sRea,sPra} \leq wva_{sd,sD,sST} \quad \text{(S355)}$$

$$wha^{Hau}_{sd,sD,sLh,sST,sRea,sPra} \leq yLh_{sLh} \quad \text{(S356)}$$

$$wha^{Hau}_{sd,sD,sLh,sST,sRea,sPra} \leq yRea_{sRea} \quad \text{(S357)}$$

$$wha^{Hau}_{sd,sD,sLh,sST,sRea,sPra} \leq yPra_{sPra} \quad \text{(S358)}$$

$$wha^{Hau}_{sd,sD,sLh,sST,sRea,sPra} \geq wva_{sd,sD,sST} + yLh_{sLh} + yRea_{sRea} + yPra_{sPra} - 3 \quad \text{(S359)}$$

$$wha^{Gni}_{sd,sD,sST,sRea} \leq wva_{sd,sD,sST} \quad \text{(S360)}$$

$$wha^{Gni}_{sd,sD,sST,sRea} \leq yRea_{sRea} \quad \text{(S361)}$$

$$wha^{Gni}_{sd,sD,sST,sRea} \geq wva_{sd,sD,sST} + yRea_{sRea} - 1 \quad \text{(S362)}$$

$$wAF_{sd,sLh,sST} \leq wA_{sd,sLh} \quad \text{(S363)}$$

$$wAF_{sd,sLh,sST} \leq yT_{sST} \quad \text{(S364)}$$

$$wAF_{sd,sLh,sST} \geq wA_{sd,sLh} + yT_{sST} - 1 \quad \text{(S365)}$$

$$\widehat{pNut}^{Hau}_{sLh,sST} = 3.66 + \frac{0{,}0668\left(\dfrac{8\,\widehat{Cp}_{sST}\,\widehat{m}_{sST}}{\pi\,\hat{k}_{sST}\,\widehat{pNB}\,\widehat{pNPt}\,\widehat{pLh}_{sLh}}\right)}{1 + 0{,}04\left(\dfrac{8\,\widehat{Cp}_{sST}\,\widehat{m}_{sST}}{\pi\,\hat{k}_{sST}\,\widehat{pNB}\,\widehat{pNPt}\,\widehat{pLh}_{sLh}}\right)^{2/3}} \quad \text{(S366)}$$

$$\widehat{pNut}^{Gni}_{tran_{sd,sST}} = \frac{0.0061\left(\dfrac{4\,\widehat{m}_{sST}}{\pi\,\hat{\mu}_{sST}\,\widehat{pdti}_{sd}\,\widehat{pNB}\,\widehat{pNPt}} - 1000\right)\left(\dfrac{\widehat{Cp}_{sST}\,\hat{\mu}_{sST}}{\hat{k}_{sST}}\right)}{1 + 12.7\left(\dfrac{0.0488}{8}\right)^{\frac{1}{2}}\left(\left(\dfrac{\widehat{Cp}_{sST}\,\hat{\mu}_{sST}}{\hat{k}_{sST}}\right)^{\frac{2}{3}} - 1\right)} \quad \text{(S367)}$$

$$\widehat{pNut}^{Gni}_{turb_{sd,sST}} = \frac{\left(0.00175 + 0.132\left(\dfrac{\pi\,\hat{\mu}_{sST}\,\widehat{pdti}_{sd}\,\widehat{pNB}\,\widehat{pNPt}}{4\,\widehat{m}_{sST}}\right)^{0.42}\right)\left(\dfrac{4\,\widehat{m}_{sST}}{\pi\,\hat{\mu}_{sST}\,\widehat{pdti}_{sd}\,\widehat{pNB}\,\widehat{pNPt}} - 1000\right)\left(\dfrac{\widehat{Cp}_{sST}\,\hat{\mu}_{sST}}{\hat{k}_{sST}}\right)}{1 + 12.7\left(0.00175 + 0.132\left(\dfrac{\pi\,\hat{\mu}_{sST}\,\widehat{pdti}_{sd}\,\widehat{pNB}\,\widehat{pNPt}}{4\,\widehat{m}_{sST}}\right)^{0.42}\right)^{1/2}\left(\left(\dfrac{\widehat{Cp}_{sST}\,\hat{\mu}_{sST}}{\hat{k}_{sST}}\right)^{\frac{2}{3}} - 1\right)} \quad \text{(S368)}$$

$$\widehat{pNua}^{Hau}_{sd,sD,sLh,sST} = 3.66 + \frac{0.0668\,\dfrac{2\,\widehat{Cp}_{sST^*}\,\widehat{m}_{sST^*}\,\widehat{pdh}^2_{sd,sD}}{\hat{k}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}\,\widehat{pNPa}\widehat{pLh}_{sLh}}}{1 + 0.04\left(\dfrac{2\,\widehat{Cp}_{sST^*}\,\widehat{m}_{sST^*}\,\widehat{pdh}^2_{sd,sD}}{\hat{k}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}\,\widehat{pNPa}\,\widehat{pLh}_{sLh}}\right)^{\frac{2}{3}}} \quad \text{(S369)}$$

$$\widehat{pNua}^{Gni}_{tran_{sd,sD,sST}} = \frac{\left(0.00337 + 4.082\left(\frac{\hat{\mu}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}\,\widehat{pNPa}}{\widehat{m}_{sST^*}\,\widehat{pdh}_{sd,sD}}\right)^{0.93}\right)\left(\frac{\widehat{m}_{sST^*}\,\widehat{pdh}_{sd,sD}}{\hat{\mu}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}\,\widehat{pNPa}} - 1000\right)\frac{\widehat{Cp}_{sST^*}\hat{\mu}_{sST^*}}{\hat{k}_{sST^*}}}{1 + 12.7\left(0.00337 + 4.082\left(\frac{\hat{\mu}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}\,\widehat{pNPa}}{\widehat{m}_{sST^*}\,\widehat{pdh}_{sd,sD}}\right)^{0.93}\right)^{\frac{1}{2}}\left(\left(\frac{\widehat{Cp}_{sST^*}\hat{\mu}_{sST^*}}{\hat{k}_{sST^*}}\right)^{\frac{2}{3}} - 1\right)} \quad \text{(S370)}$$

$$\widehat{pNua}^{Gni}_{turb_{sd,sD,sST}} = \frac{\left(0.02225\left(\frac{\hat{\mu}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}\,\widehat{pNPa}}{\widehat{m}_{sST^*}\,\widehat{pdh}_{sd,sD}}\right)^{0.1865}\right)\left(\frac{\widehat{m}_{sST^*}\,\widehat{pdh}_{sd,sD}}{\hat{\mu}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}\,\widehat{pNPa}} - 1000\right)\frac{\widehat{Cp}_{sST^*}\hat{\mu}_{sST^*}}{\hat{k}_{sST^*}}}{1 + 12.7\left(0.02225\left(\frac{\hat{\mu}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}\,\widehat{pNPa}}{\widehat{m}_{sST^*}\,\widehat{pdh}_{sd,sD}}\right)^{0.1865}\right)^{\frac{1}{2}}\left(\left(\frac{\widehat{Cp}_{sST^*}\hat{\mu}_{sST^*}}{\hat{k}_{sST^*}}\right)^{\frac{2}{3}} - 1\right)} \quad \text{(S371)}$$

where if $sST = h$, then $sST^* = c$ and vice-versa.

Bounds on flow velocity and pressure drop are still given by Eqs. (S206) to (S211). The objective function is given by:

$$\min A \quad \text{(S372)}$$

Section S3

This section recaptures the problem data for Examples 1 to 4 from Peccini et al (2019). The stream data are shown in Table S1 and Table S2.

Table S1. Stream data for Examples 1 to 4.

Parameter	Unit	Example 1		Example 2		Example 3		Example 4	
		Hot stream	Cold stream	Hot stream	Cold stream	Hot stream	Cold stream	Hot stream	Cold stream
Mass flow, $\hat{m}_{sST}$	kg/s	1.22	1.26	2.11	2.52	16.5	34.32	25.9	13.14
Inlet temperature, $\widehat{Ti}_{sST}$	°C	65.55	15.55	60	20	90	30.1	60	10
Outlet temperature, $\widehat{To}_{sST}$	°C	37.75	48.85	50	30	50	40	50	25
Density, $\hat{\rho}_{sST}$	kg/m³	1022	879	1000	850	786	995	780	1050
Viscosity, $\hat{\mu}_{sST}$	Pa·s	2×10^{-3}	5.5×10^{-4}	2×10^{-4}	5.5×10^{-4}	1.89×10^{-3}*	7.2×10^{-4}	9.5×10^{-4}	0.024
Heat capacity, $\widehat{Cp}_{sST}$	J/(kg°C)	2,177.14	1,758.46	2,100	1,760	2,177	4,187	1,900	2,500
Thermal conductivity $\hat{k}_{sST}$	W/(m°C)	0.173	0.159	0.175	0.160	0.12	0.59	0.18	0.264
Fouling resistance, $\widehat{Rf}_{sST}$	m²°C/W	1.76×10^{-4}	1.76×10^{-4}	2×10^{-4}	2×10^{-4}	2×10^{-4}	4×10^{-4}	3×10^{-4}	3×10^{-4}
Available pressure drop, $\widehat{\Delta P}_{sST\ disp}$	kPa	138	138	50	50	100	100	150	150

*The original article shows 1.89×10^{-4} due to a typing error.

The allowed flow velocity for both sides is between 1 and 3 m/s; The pipe thermal conductivity is 16.27 W/(m °C) for Example 1 and 55 W/(m °C) for Examples 2 to 4; The minimum excess area is 10% for Examples 1 and 4 and 20% for Examples 2 and 3.

Table S2. Search domain from Peccini et al (2019).

Parameter	Unit	Discrete options								
$\widehat{pNB}_{sB}$	--	1 to 20								
$\widehat{pNE}_{sE}$	--	1 to 20								
$\widehat{pNh}_{sNh}$	--	1 to 10								
$\widehat{pL}h_{sLh}$	ft	5	10	15	20	25				
$\widehat{pL}h_{sLh}$	m	1.524	3.048	4.572	6.096	7.620				
Inner tube NPS*	in	½	¾	1	1 ¼	1 ½	2	2 ½	3	3 ½
Inner tube OD‡	m	0.021	0.027	0.033	0.042	0.048	0.060	0.073	0.089	0.102
Outer tube NPS*	in	1 ¼	1 ½	2	2 ½	3	3 ½	4	4 ½	5
Outer tube OD‡	m	0.042	0.048	0.060	0.073	0.089	0.102	0.114	0.127	0.168

Section S4

This section shows a comparison between the performance of models Par-GEO-MILP, Par-FLU-MILP and Par-STR-MILP.

The four examples (see Table S1) display very different features, in terms of its streams properties and heat task magnitude, with heat transfer rates ranging from 44.3 kW to 1,436.8 kW (32 times higher). To cover all these different features and show the modular flexibility of the double pipe heat exchanger a broad search domain (discrete available options for the decision variables) was proposed for these examples, as shown in Table S2.

A great advantage of a MILM solved using MILP approaches, besides not requiring initial estimates, is the guarantee of global optimality. A drawback, however, is that both the number of variables and the number of constraints increase significantly, due to the mathematical reformulation. Table S3 shows the problem dimension for the original search domain (Table S2) for a MINLP approach of Peccini et al (2019) and for the MILP here proposed.

Table S3. Increase in the problem dimension (Raw MINLP × MILP).

	Raw MINLP	MILP
Number of constraints	121	1,467,771,040
Number of variables	180	300,784,517
Number of discrete variables	111	111

The increase in the problem dimension was so high that computational memory drawbacks were encountered, and the problems could not be directly solved with CPLEX (GAMS Software 24.7.1, ran on an AMD Ryzen 9 3900X 12-Core Processor).

A few mitigating strategies were proposed, but to show the problems, the search space (the number of discrete variables) was temporarily restricted (Table S4).

Table S4. Reduced search domain for computational issues discussion.

Parameter	Unit	Discrete options Example 3			
$\widehat{pNB}_{sB}$	--	1 to 20			
$\widehat{pNE}_{sE}$	--	1 to 3			
$\widehat{pNh}_{sNh}$	--	1 to 6			
$\widehat{pLh}_{sLh}$	ft	15	20	25	
$\widehat{pLh}_{sLh}$	m	4.572	6.096	7.620	
Inner tube NPS	in	1	1 ¼	1 ½	2
Inner tube OD ($\widehat{pdte}_{sd}$)	m	0.033	0.042	0.048	0.060
Outer tube NPS	in	1 ½	2	2 ½	3
Outer tube OD ($\widehat{pDte}_{sD}$)	m	0.048	0.060	0.073	0.089

To discuss these strategies, we start with Example 3, which is the most critical example in terms of computational issues (it will become clearer why further on the discussion). The new problem sizes, for both MINLM and MILM, as well as the computational effort required (for Example 3), are shown in Table S5. The elapsed time was obtained through Gams Utility and Performance Function *timeElapsed*, which returns the elapsed time since the start of a GAMS run in seconds. The solver time was obtained through Gams Option *EtSolver*, which is the elapsed time taken by the solver only.

Table S5. Increase in the problem dimension for a reduced search domain – Example 3 (Raw MINLP × MILP).

	Raw MINLP	MILP
Number of constraints	103	2,836,150
Number of variables	130	599,577
Number of discrete variables	61	61
Elapsed time (s)	76.04	5,894.34
Solver time (s)	76.02	5,888.51

When comparing the MILP model with the MINLP model, the number of discrete variables does not change, since it is related to the search domain and available structural options which are the same for both. Meanwhile, for the number of equations and continuous variables there is a steep increase, in the order of tens of thousands. Inevitably, it also impacts on the computational effort required for the optimization, which also increased significantly for the MILP model (77 times). Although this shortcoming becomes less and less relevant with the constant growth and progress of the available computation tools, we propose alternatives to reduce the computational effort and the memory requirements.

In order reduce the number of variables and constraints, enabling the problem to be solved faster and with less memory requirement, one may rewrite the MILM with a given set of the decision variables turned into parameters. This reduces the number of discrete variables and the number of auxiliary variables and constraints generated during the reformulation. The new parameterized MILM is solved using a subset of the search domain of the original problem. To solve the entire problem, one may repeat the optimization sequentially to cover the entire domain or using a more efficiency search procedure (we present alternatives in the paper).

To further clarify the idea, let us classify the binary decision variables in three categories: geometric (yd_{sd}, yD_{sD} and yLh_{sLh}), structural (yNB_{sB}, yNh_{sNh}, yPt_{sE} and yPa_{sE}) and fluid allocation (yT_{sST}). One can use the geometric discrete variables (*dte, dti, Dte, Dti, Lh*) as known parameters, therefore eliminating some nonlinearities in the original MINLM, and consequently eliminating a significant number of variables and constraints. Solving this new parametrized MILM with a MILP approach renders the best possible structure arrangement and fluid allocation for a given hairpin dimension. Solving sequentially for each possible combination of hairpin dimension, the best solution encountered is the global optimum for the original problem.

To assess this idea, three parametrized MILP were generated (Par-GEO-MILP, Par-FLU-MILP and Par-STR-MILP) and they are detailed in Section S2 of this document. The variables transformed into parameters were the geometric ones for the first parametrized MILP, both the dimension and fluid allocation for the second and the structural ones for the third one. Table S6 shows the number of variables and constraints (all for the reduced search space show in Table S4). The parametrized MILP were solved sequentially using exhaustive enumeration of all possible alternatives.

Table S6. Exhaustive Enumeration - Problem dimensions for MILP alternatives with reduced search domain – Example 3.

	Original MILP	Par-GEO-MILP	Par-FLU-MILP	Par-STR-MILP
Number of constraints	2,836,150	88,295	33,896	5,791
Number of variables	599,577	22,406	10,634	1,355
Number of discrete variables	61	50	44	29
Average solver time for a single optimization run (s)	5,888.51	2.03	0.50	0.335
Number of enumerated options solved	1	36	72	600
Total solver time (s)	5,888.51	73.0	35.9	201.2
Total elapsed time (s)	5,894.34	77.52	39.0	216.39

A significant reduction of the number of variables and constraints takes place: around 96,5% for Par-GEO-MILP, 98,8% for Par-FLU-MILP and 99,8% for Par-STR-MILP. The computational time for one optimization run of the parametrized MILMs is significantly smaller: averages of 2.03s for the Par-GEO-MILP, 0.50s for Par-FLU-MILP and 0.33s for Par-STR-MILP.

Although a single run of Par-STR-MILP has the fastest solver time, the number of parameter combinations for the exhaustive enumerations (600) is larger than for the Par-GEO-MILP (36) and for Par-FLU-MILP (72), therefore the total computational time required becomes higher than the other two Par-MILP. All three showed a significant time reduction when compared to the original MILP: 98.75 % for Par-GEO-MILP, 99.4% for Par-FLU-MILP and 96.6 % for Par-STR-MILP.

After these promising results, we return to the original domain (Table S2). The problem dimensions for all proposed Par-MILP are depicted in Table S7.

Table S7. Exhaustive Enumeration - Problem dimensions for MILP alternatives for the original search domain – Example 3.

	Original MILP	Par-GEO-MILP	Par-FLU-MILP	Par-STR-MILP
Number of constraints	1,467,771,040	6,048,095	2,336,856	40,560
Number of variables	300,784,517	1,532,124	725,712	9,124
Number of discrete variables	111	88	82	41
Average solver time for a single optimization run (s)	–	–	25.5	0.60
Number of enumerated options solved	–	–	590	7,800
Solver time (s)	–	–	15,042.2	4,704.5
Elapsed time (s)	–	–	17,738.9	5,357.4

Par-GEO-MILP, associated with a larger number of variables and constraints presented, once again, memory limitations (which is why no computational time is reported). The exhaustive enumeration was successfully applied for the original domain for both Par-FLU-MILP and Par-STR-MILP. The latter, although associated with a larger number of possible combinations for the parametrized set (7,800), has a smaller problem dimension, which is associated to a much smaller single run average solver time (0.6s), resulting in a total solver time of 1h 18min. Par-FLU-MILP, despite its promising results for the reduced domain and the smaller number of enumeration options when compared to Par-STR-MILP, is associated to a great increase in the number of variables and constraints, which results in significantly slower single runs (average computational time of 25.5s compared to 0.6s for Par-STR-MILP). So, for the proposed search domain, Par-STR-MILP was the most suitable model. However, it is important to notice that if the search domain for a given problem has less structure combinations, but more hairpin dimension combinations, another MILP alternative could be the most suitable one.

Section S5

This section shows the details of the globally optimal solutions for Examples 1 to 4.

Table S8. Globally optimal solutions for Examples 1 to 4.

Variable	Unit	Example 1	Example 2	Example 3	Example 4
Tube side stream	–	Hot stream	Cold stream	Hot stream	Cold stream
Total heat exchanger area, A	m²	9.19	1.84	88.73	40.86
Required heat exchanger area	m²	7.52	1.51	73.94	37.01
Inner tube NPS – OD	in – m	¾ – 0.027	½ – 0.021	1 ½ – 0.048	3 ½ – 0.102
Outer tube NPS – OD	in – m	1 ¼ – 0.042	1 ¼ – 0.042	2 ½ – 0.073	4 ½ – 0.127
Hairpin length, Lh	ft – m	15 – 4.572	15 – 4.572	20 – 6.096	10 – 3.048
Number of hairpins per unit, pNh	–	8	1	6	7
Number of branches, pNB	–	3	2	8	1
Tube side units in parallel, $pNPt$	–	1	3	1	1
Annular side units in parallel, $pNPa$	–	1	1	2	6
Tube side velocity, vt	m/s	1.16	2.52	2.00	1.96
Annulus side velocity, va	m/s	1.18	1.74	1.71	2.54
Tube side film coefficient, ht	W/(m²°C)	1384.5	4,292	1,397	656
Annulus side film coefficient, ha	W/(m²°C)	2235.2	6,046	9,129	3,276
Overall heat transfer coefficient, U	W/(m²°C)	508.4	991. 6	601.3	360.6
Correction Factor, F	–	1	0.989	0.979	0.986
Tube side pressure drop, dPt	kPa	40.8	19.0	76.3	110.7
Annulus side pressure drop, dPa	kPa	77.8	30.3	93.7	110.8

The respective structures are illustrated in Figure S1.

The different services of the four examples are evidenced by the different optimal solutions, in terms of structure and of heat exchanger areas (that range from 1.84m² to 88.73m²). The solutions found are guaranteed global optimums.

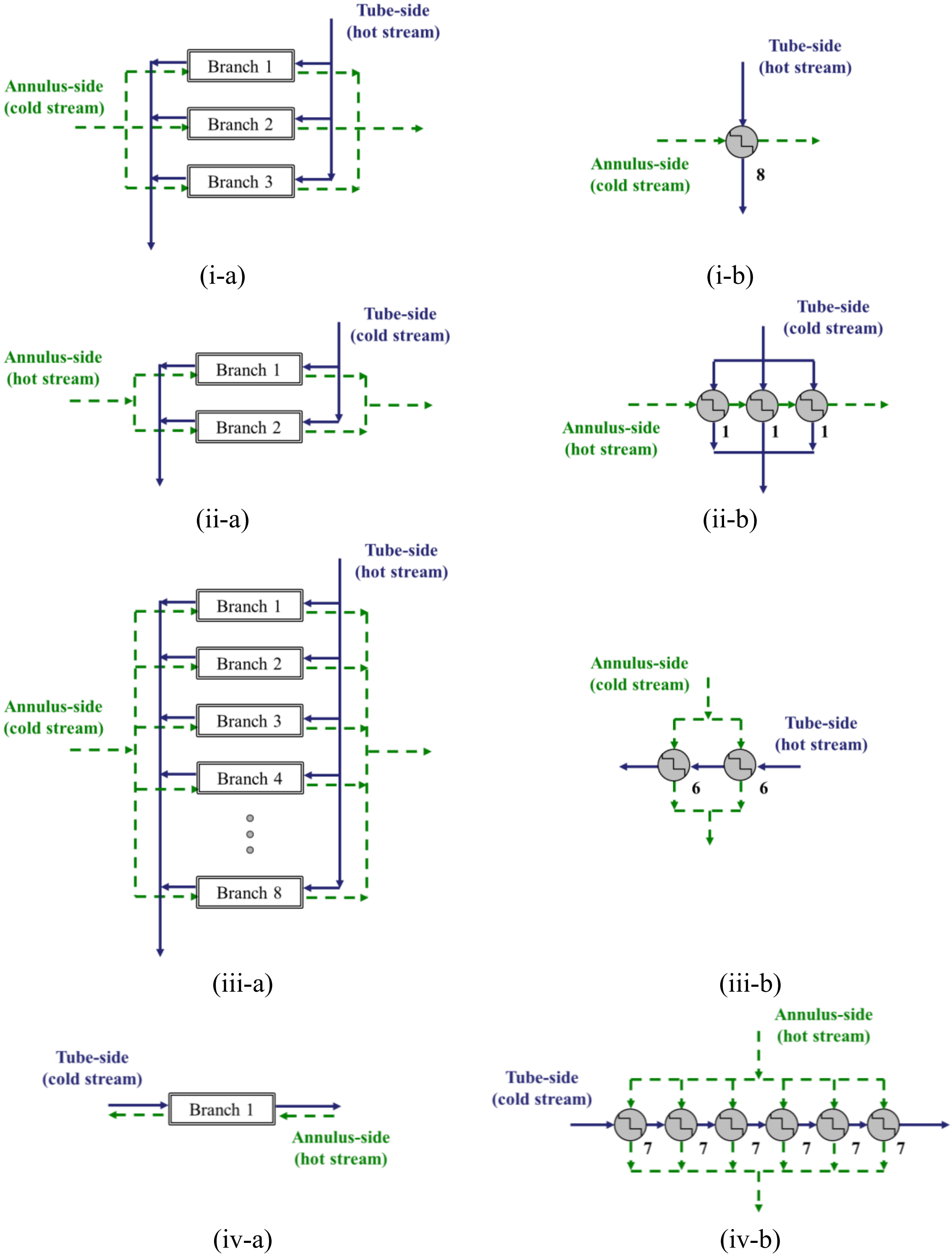

Figure S1. Globally optimal solutions: (i) Example 1; (ii) Example 2; (iii) Example 3; (iv) Example 4; (a) General structure; (b) branch structure.

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1. Heat exchanger design equations

The basic structure of double pipe heat exchangers consists in two concentric tubes. The heat transfer equations, based on the LMTD method, are:

$$\hat{Q} = \hat{m}_h Cp_h(\widehat{Ti}_h - \widehat{To}_h) \quad (1)$$

$$\hat{Q} = \hat{m}_c Cp_c(\widehat{To}_c - \widehat{Ti}_c) \quad (2)$$

$$\hat{Q} = UA\widehat{\Delta Tlm} \quad (3)$$

$$\frac{1}{U} = \frac{1}{h_t}\frac{d_e}{d_i} + \widehat{Rf}_t\frac{d_e}{d_i} + \frac{d_e \ln\left(\frac{d_e}{d_i}\right)}{2\widehat{k_{tube}}} + \widehat{Rf}_a + \frac{1}{h_a} \quad (4)$$

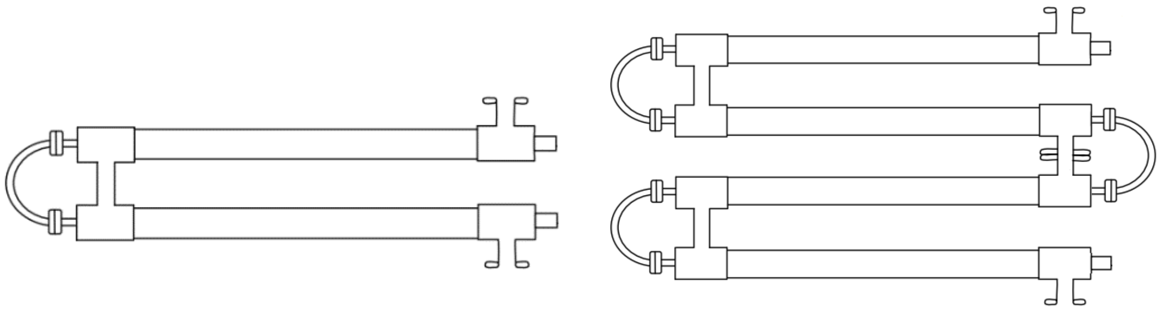
The heat transfer coefficients (h_t, h_a) are obtained from a Nusselt number correlation (we use Gnielinski et al., Hauser et al. and Sieder & Tate) (Incropera and Dewitt, 2007). Finally, ignoring minor head losses in connections and bends, the pressure drop of the flow in the inner tube is calculated by the Darcy-Weisbach equation. In turn, the annular section pressure drop is obtained using the same equation, but using hydraulic diameters (omitting the viscosity correction factor) (Saunders, 1988).

$$\Delta P_t = \rho_t \, f_t \frac{L}{d_i}\frac{v_t^2}{2} \quad (5)$$

$$\Delta P_a = \rho_a \, f_a \frac{L}{dh}\frac{v_a^2}{2} \quad (6)$$

2. Double pipe heat exchanger hairpin architecture

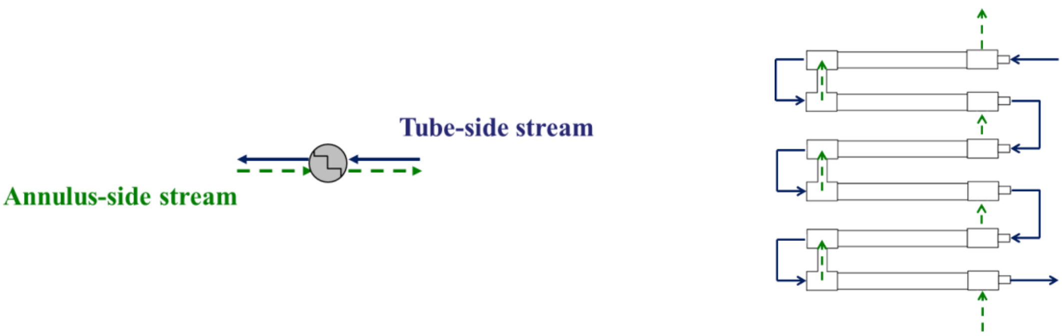
Double pipe heat exchangers are usually commercialized in hairpin structures, as shown in Figure 2a. Figure 2b illustrates two hairpins connected in series.



(a) Hairpin structure (b) Two hairpins connected in series.

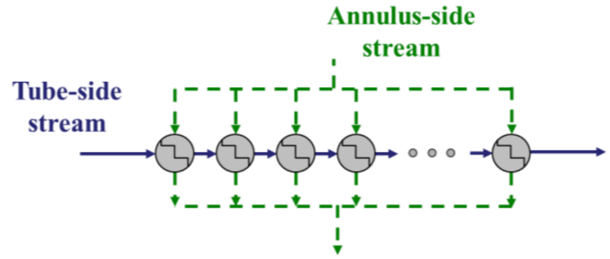
Figure 2. Double pipe heat exchanger hairpin structure.

Different interconnection patterns among the heat exchanger hairpins provide flexible alternatives to abide by heat load and maximum pressure drop specifications of the service. In our model, a hairpin is the countercurrent basic structure; a unit is defined as multiple hairpins connected in series; a branch is a structure that can be arranged in some proposed complex series/parallel configurations (see Figure 3), which can then be arranged in a set of parallel branches, rendering the general structure (See Figure 4) (Peccini et al., 2019).

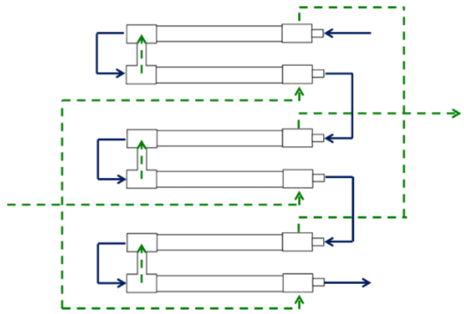


(i-a) Structure – Type I

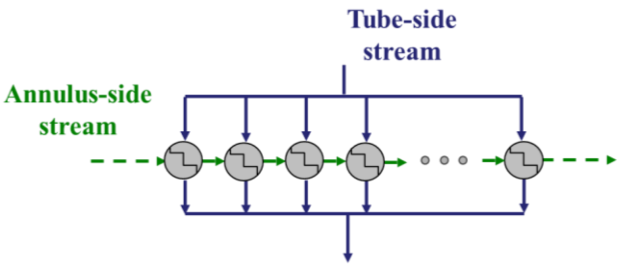
(i-b) Specific example of one unit (three hairpins per unit)



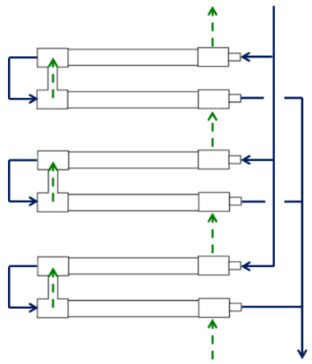
(ii-a) Structure – Type II



(ii-b) Specific example of three units (one hairpin per unit)

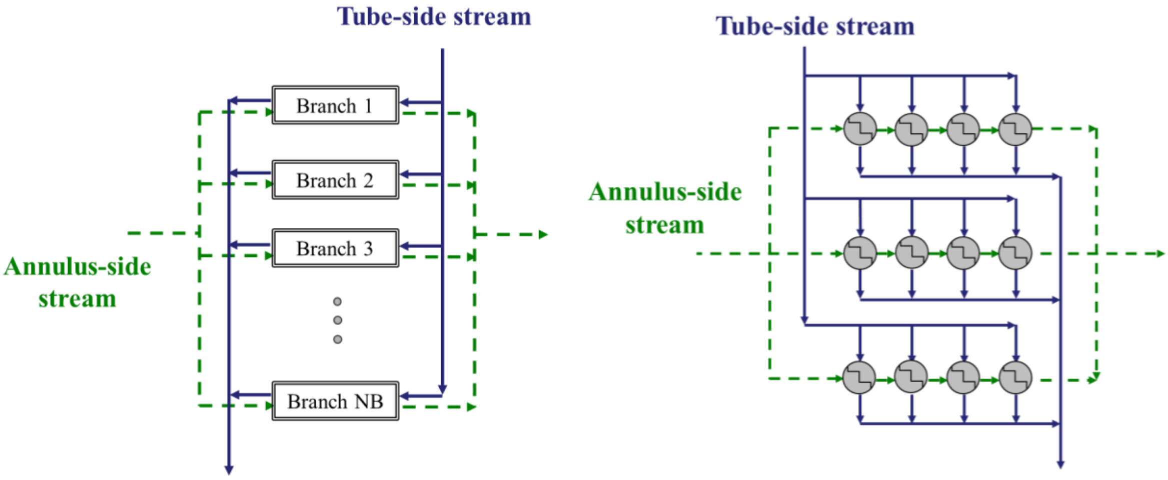


(iii-a) Structure – Type III



(iii-b) Specific example of three units (one hairpin per unit)

Figure 3. Different flow arrangements – (i) Type I: tube-side and annulus-side streams in series; (ii) Type II: tube-side stream in series and annulus-side stream in parallel; (iii) Type III: annulus-side stream in series and tube-side stream in parallel.



(a) General Structure

(b) Specific example with three branches (type III) (four units per branch)

Figure 4. Multiple parallel branches.

3. Linear formulation of the design problem

In a prior work (Peccini et al., 2019), a MINLM was proposed (see Supporting Information - Section S1). Here, we show that this model can be rigorously reformulated in a linear one. The techniques employed in this transformation are described in Costa and Bagajewicz (2019).

3.1. Selection of the geometric variables

We use the following set of binaries to describe the design variables: yd_{sd} for the inner tube diameter (discrete values of the outer and inner values: $\widehat{pdte}_{sd}$ and $\widehat{pdti}_{sd}$), yD_{sD} for the outer tube diameter (discrete values of the outer and inner values: $\widehat{pDte}_{sd}$ and $\widehat{pDti}_{sd}$), yLh_{sLh} for the hairpin tube length (discrete values: $\widehat{pLh}_{sLh}$), yNh_{sNh} for the number of hairpins per unit (discrete values: $\widehat{pNh}_{sNh}$), yB_{sB} for the number of parallel branches present in the heat exchanger design (discrete values: $\widehat{pNB}_{sB}$), yPt_{sE} and yPa_{sE} for the number of units aligned in parallel in each branch for the tube-side and the annulus-side streams (discrete values: $\widehat{pNE}_{sE}$).

This set of binary variables must be associated to constraints to ensure that only one of the available options will be selected:

$$\sum_{sd=1}^{sdmax} yd_{sd} = \sum_{sD=1}^{sDmax} yD_{sD} = \sum_{sLh=1}^{sLhmax} yLh_{sLh} = \sum_{sNh=1}^{sNhmax} yNh_{sNh} = \sum_{sB=1}^{sBmax} yB_{sB} = \sum_{sE=1}^{sEmax} yPt_{sE} = \sum_{sE=1}^{sEmax} yPa_{sE} = 1 \qquad (7)$$

3.2. Stream allocation

The stream allocation is controlled by the binary variables yTc and yTh. If $yTc = 1$, then the cold stream flows inside the inner tube, otherwise, $yTh = 1$, which is guaranteed by the following constraint:

$$yT_c + yT_h = 1 \qquad (8)$$

3.3. Structural constraints

The following constraint ensures that if the tube-side has more than one parallel passage, the annular side can be only arranged in series and vice-versa:

$$yPt_{sE=1} + yPa_{sE=1} \geq 1 \qquad (9)$$

The following constraint guarantees that the outer tube inner diameter is larger than the inner tube outer diameter.

$$yd_{sd} + yD_{sD} \leq 1 \qquad \forall\ (sd, sD)\ \in SDD \qquad (10)$$

where SDD is the set of forbitten (sd,sD) combinations.

3.4. Inner tube thermal and hydraulic modeling

The flow velocity inside the inner tube, its corresponding Reynolds number and additional linear inequalities are:

$$vt = \sum_{sd=1}^{sdmax} \sum_{sB=1}^{sBmax} \sum_{sE=1}^{sEmax} \sum_{sST} \frac{4\,\widehat{m}_{sST}}{\pi\,\hat{\rho}_{sST}\widehat{pdti}_{sd}^{2}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}\, wvt_{sd,sB,sE,sST} \tag{11}$$

$$Ret = \sum_{sd=1}^{sdmax} \sum_{sB=1}^{sBmax} \sum_{sE=1}^{sEmax} \sum_{sST} \frac{4\,\widehat{m}_{sST}}{\pi\,\hat{\mu}_{sST}\widehat{pdti}_{sd}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}\, wvt_{sd,sB,sE,sST} \tag{12}$$

$$wvt_{sd,sB,sE,sST} \le yd_{sd} \tag{13}$$

$$wvt_{sd,sB,sE,sST} \le yB_{sB} \tag{14}$$

$$wvt_{sd,sB,sE,sST} \le yPt_{sE} \tag{15}$$

$$wvt_{sd,sB,sE,sST} \le yT_{sST} \tag{16}$$

$$wvt_{sd,sB,sE,sST} \ge yd_{sd} + yB_{sB} + yPt_{sE} + yT_{sST} - 3 \tag{17}$$

The Prandtl number of the inner tube stream becomes:

$$Prt = \frac{\widehat{Cp}_c\hat{\mu}_c}{\hat{k}_c}\, yT_c + \frac{\widehat{Cp}_h\hat{\mu}_h}{\hat{k}_h}\, yT_h \tag{18}$$

The evaluation of the Nusselt number by the Sieder & Tate correlation is now:

$$Nut^{S\&T} = \sum_{sB=1}^{sBmax} \sum_{sE=1}^{sEmax} \sum_{sLh=1}^{sLhmax} \sum_{sST} \widehat{pNut}_{sB,sE,sLh,sST}^{S\&T} wNut_{sB,sE,sLh,sST} \tag{19}$$

$$wNut_{sB,sE,sLh,sST} \le yB_{sB} \tag{20}$$

$$wNut_{sB,sE,sLh,sST} \le yPt_{sE} \tag{21}$$

$$wNut_{sB,sE,sLh,sST} \le yLh_{sLh} \tag{22}$$

$$wNut_{sB,sE,sLh,sST} \le yT_{sST} \tag{23}$$

$$wNut_{sB,sE,sLh,sST} \ge yB_{sB} + yPt_{sE} + yLh_{sLh} + yT_{sST} - 3 \tag{24}$$

where the parameter $\widehat{pNut}_{sB,sE,sLh,sST}^{S\&T}$, is given by:

$$\widehat{pNut}^{S\&T}_{sB,sE,sLh,sST} = 1.86\left(\frac{8\ \widehat{Cp}_{sST}\ \widehat{m}_{sST}}{\pi\ \hat{k}_{sST}\ \widehat{pNB}_{sB}\ \widehat{pNE}_{sE}\ \widehat{pLh}_{sLh}}\right)^{1/3} \quad (25)$$

The following equation allows the selection of the appropriate Re number using binary variables.

$$Ret \ \leq 1311\ yRet_1 + 2300\ yRet_2 + 3380\ yRet_3 + \widehat{URe}\ yRet_4 \quad (26)$$

$$Ret \ \geq 1311 yRet_2 + 2300 yRet_3 + 3380\ yRet_4 + \varepsilon \quad (27)$$

$$Prt \ \leq 5 yPrt_1 + \widehat{UPr}\ yPrt_2 \quad (28)$$

$$Prt \ \geq 5 yPrt_2 + \varepsilon \quad (29)$$

$$Nut^{S\&T} \leq 3{,}66 yNut_1 + \widehat{UNu}\ yNut_2 - \varepsilon \quad (30)$$

$$Nut^{S\&T} \geq 3{,}66 yNut_2 \quad (31)$$

$$\sum_{sRet=1}^{sRetmax} yRet_{sRet} = 1 \quad (32)$$

$$yPrt_1 + yPrt_2 = 1 \quad (33)$$

$$yNut_1 + yNut_2 = 1 \quad (34)$$

The pressure drop in the tubes becomes:

$$\begin{aligned} \Delta Pt \\ = \sum_{sd=1}^{sdmax}\sum_{sB=1}^{sBmax}\sum_{sNh=1}^{sNhmax}\sum_{sE=1}^{sEmax}\sum_{sLh=1}^{sLhmax}\sum_{sE'=1}^{sEmax}\sum_{sST}\Big(&\widehat{pdPt}1_{sd,sB,sE,sNh,sLh,sE',sST}\, wdPt_{sd,sB,sNh,sLh,sE,sE',sST,1} \\ &+ \widehat{pdPt}23_{sd,sB,sE,sNh,sLh,sE',sST}\, wdPt_{sd,sB,sNh,sLh,sE,sE',sST,2} \\ &+ \widehat{pdPt}23_{sd,sB,sE,sNh,sLh,sE',sST}\, wdPt_{sd,sB,sNh,sLh,sE,sE',sST,3} \\ &+ \widehat{pdPt}4_{sd,sB,sE,sNh,sLh,sE',sST}\, wdPt_{sd,sB,sNh,sLh,sE,sE',sST,4}\Big) \end{aligned} \quad (35)$$

$$wdPt_{sd,sB,sE,sNh,sLh,sE',sST,sRet} \leq wA_{sd,sB,sE,sNh,sLh,sE'} \quad (36)$$

$$wdPt_{sd,sB,sE,sNh,sLh,sE',sST,sRet} \leq yT_{sST} \quad (37)$$

$$wdPt_{sd,sB,sE,sNh,sLh,sE',sST,sRet} \leq yRet_{sRet} \quad (38)$$

$$wdPt_{sd,sB,sE,sNh,sLh,sE',sST,sRet} \geq wA_{sd,sB,sE,sNh,sLh,sE'} + yT_{sST} + yRet_{sRet} - 2 \tag{39}$$

The additional parameters inserted for simplification purposes in Eq. (35) are given by:

$$\widehat{pdPt}1_{sd,sB,sE,sNh,sLh,sE',sST} = \frac{128\ \hat{\mu}_{sST}\ \widehat{m}_{sST}\widehat{pNh}_{sNh}\widehat{pLh}_{sLh}\widehat{pNE}_{sE'}}{\pi\ \hat{\rho}_{sST}\ \widehat{pdti}^{4}_{sd}\widehat{pNB}_{sB}\widehat{pNE}_{sE}} \tag{40}$$

$$\widehat{pdPt}23_{sd,sB,sE,sNh,sLh,sE',sST} = \frac{0.3904\ \widehat{m}^{2}_{sST}\widehat{pNh}_{sNh}\widehat{pLh}_{sLh}\widehat{pNE}_{sE'}}{\pi^{2}\ \hat{\rho}_{sST}\ \widehat{pdti}^{5}_{sd}\widehat{pNB}^{2}_{sB}\widehat{pNE}^{2}_{sE}} \tag{41}$$

$$\widehat{pdPt}4_{sd,sB,sE,sNh,sLh,sE',sST} = \frac{0.112\ \widehat{m}^{2}_{sST}\widehat{pNh}_{sNh}\widehat{pLh}_{sLh}\widehat{pNE}_{sE'}}{\pi^{2}\ \hat{\rho}_{sST}\ \widehat{pdti}^{5}_{sd}\widehat{pNB}^{2}_{sB}\widehat{pNE}^{2}_{sE}} + \frac{4.719\ \hat{\mu}^{0.42}_{sST}\ \widehat{m}^{1.58}_{sST}\widehat{pNh}_{sNh}\widehat{pLh}_{sLh}\widehat{pNE}_{sE'}}{\pi^{1.58}\ \hat{\rho}_{sST}\ \widehat{pdti}^{4.58}_{sd}\widehat{pNB}^{1.58}_{sB}\widehat{pNE}^{1.58}_{sE}} \tag{42}$$

3.5. Annulus side thermal and hydraulic modeling

The annulus-side stream velocity, its corresponding Reynolds number and additional linear inequalities are given by:

$$va = \sum_{sd=1}^{sdmax}\sum_{sD=1}^{sDmax}\sum_{sB=1}^{sBmax}\sum_{sE=1}^{sEmax}\sum_{sST}\frac{\widehat{m}_{sST^*}}{\hat{\rho}_{sST^*}\ \widehat{pAa}_{sd,sD}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}wva_{sd,sD,sB,sE,sST} \tag{43}$$

$$Rea = \sum_{sd=1}^{sdmax}\sum_{sD=1}^{sDmax}\sum_{sB=1}^{sBmax}\sum_{sE=1}^{sEmax}\sum_{sST}\frac{\widehat{m}_{sST^*}\ \widehat{pdh}_{sd,sD}}{\hat{\mu}_{sST^*}\ \widehat{pAa}_{sd,sD}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}wva_{sd,sD,sB,sE,sST} \tag{44}$$

$$wva_{sd,sD,sB,sE,sST} \leq yd_{sd} \tag{45}$$

$$wva_{sd,sD,sB,sE,sST} \leq yD_{sD} \tag{46}$$

$$wva_{sd,sD,sB,sE,sST} \leq yB_{sB} \tag{47}$$

$$wva_{sd,sD,sB,sE,sST} \leq yPa_{sE} \tag{48}$$

$$wva_{sd,sD,sB,sE,sST} \leq yT_{sST} \tag{49}$$

$$wva_{sd,sD,sB,sE,sST} \geq yd_{sd} + yD_{sD} + yB_{sB} + yPa_{sE} + yT_{sST} - 4 \tag{50}$$

where if $sST = h$, then $sST^* = c$ and vice-versa.

The Prandtl number and the Nusselt numbers for the annulus-side stream becomes:

$$Pra = \frac{\widehat{Cp}_c \hat{\mu}_c}{\hat{k}_c}\, yT_h + \frac{\widehat{Cp}_h \hat{\mu}_h}{\hat{k}_h}\, yT_c \tag{51}$$

$$Nua^{S\&T} = \sum_{sd=1}^{sdmax} \sum_{sD=1}^{sDmax} \sum_{sB=1}^{sBmax} \sum_{sE=1}^{sEmax} \sum_{sLh=1}^{sLhmax} \sum_{sST} \widehat{pNua}^{S\&T}_{sd,sD,sB,sE,sLh,sST}\, wNua_{sd,sD,sB,sE,sLh.sST} \tag{52}$$

$$wNua_{sd,sD,sB,sE,sLh.sST} \leq wva_{sd,sD,sB,sE,sST} \tag{53}$$

$$wNua_{sd,sD,sB,sE,sLh.sST} \leq yLh_{sLh} \tag{54}$$

$$wNua_{sd,sD,sB,sE,sLh.sST} \geq wva_{sd,sD,sB,sE,sST} + yLh_{sLh} - 1 \tag{55}$$

where the parameter $\widehat{pNua}^{S\&T}_{sd,sD,sB,sE,sLh,sST}$ is given by:

$$\widehat{pNua}^{S\&T}_{sd,sD,sB,sE,sLh,sST} = 1.86\left(\frac{2\,\widehat{Cp}_{sST^*}\,\widehat{m}_{sST^*}\,\widehat{pdh}^2_{sd,sD}}{\hat{k}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}_{sB}\widehat{pNE}_{sE}\widehat{pLh}_{sLh}}\right)^{\frac{1}{3}} \tag{56}$$

where if $sST = h$, then $sST^* = c$ and vice-versa.

The constraints relating binary variables to Re ranges for the tube-side remain the same:

$$Rea \leq 500\, yRea_1 + 2300\, yRea_2 + 10000\, yRea_3 + \widehat{URe}\, yRea_4 \tag{57}$$

$$Rea \geq 500 yRea_2 + 2300 yRea_3 + 10000\, yRea_4 + \varepsilon \tag{58}$$

$$Pra \leq 5yPra_1 + \widehat{UPr}\, yPra_2 \tag{59}$$

$$Pra \geq 5yPra_2 + \varepsilon \tag{60}$$

$$Nua^{S\&T} \leq 3.66 yNua_1 + \widehat{UNu}\, yNua_2 - \varepsilon \tag{61}$$

$$Nua^{S\&T} \geq 3.66 yNua_2 \tag{62}$$

$$\sum_{sRea=1}^{sReamax} yRea_{sRea} = 1 \tag{63}$$

$$yPra_1 + yPra_2 = 1 \tag{64}$$

$$yNua_1 + yNua_2 = 1 \tag{65}$$

The pressure drop of the flow in the annulus is given by

$$\Delta Pa = \sum_{sd=1}^{sdmax}\sum_{sD=1}^{sDmax}\sum_{sB=1}^{sBmax}\sum_{sE=1}^{sEmax}\sum_{sNh=1}^{sNhmax}\sum_{sLh=1}^{sLhmax}\sum_{sE'=1}^{sEmax}\sum_{sST}(\widehat{pdPa}1_{sd,sD,sB,sE,sNh,sLh,sE',sST} wdPa_{sd,sD,sB,sNh,sLh,sE.sE'sST,1} + \widehat{pdPa}23_{sd,sD,sB,sE,sNh,sLh,sE',sST}(wdPa_{sd,sD,sB,sNh,sLh,sE.sE'sST,sRea} + wdPa23_{sd,sD,sB,sNh,sLh,sE.sE'sST,3}) + \widehat{pdPa}4_{sd,sD,sB,sE,sNh,sLh,sE',sST} wdPa_{sd,sD,sB,sNh,sLh,sE.sE'sST,4}) \tag{66}$$

$$wdPa_{sd,sD,sB,sE,sNh,sLh,sE',sST,sRea} \leq wA_{sd,sB,sE,sNh,sLh,sE'} \tag{67}$$

$$wdPa_{sd,sD,sB,sNh,sLh,sE.sE'sST,sRea} \leq yD_{sD} \tag{68}$$

$$wdPa_{sd,sD,sB,sNh,sLh,sE.sE'sST,sRea} \leq yT_{sST} \tag{69}$$

$$wdPa_{sd,sD,sB,sNh,sLh,sE.sE'sST,sRea} \leq yRea_{sRea} \tag{70}$$

$$wdPa_{sd,sD,sB,sNh,sLh,sE.sE'sST,sRea} \geq wA_{sd,sB,sNh,sLh,sE,sE'} + yD_{sD} + yT_{sST} + yRea_{sRea} - 3 \tag{71}$$

where:

$$\widehat{pdPa}1_{sd,sD,sB,sE,sNh,sLh,sE',sST} = \frac{32\,\hat{\mu}_{sST^*}\,\hat{m}_{sST^*}\,\widehat{pNh}_{sNh}\widehat{pLh}_{sLh}\widehat{pNE}_{sE}}{\hat{\rho}_{sST^*}\,\widehat{pdh}^2_{sd,sD}\,\widehat{pAa}_{sd,sD}\widehat{pNB}_{sB}\,\widehat{pNE}_{sE'}} \tag{72}$$

$$\widehat{pdPa}23_{sd,sD,sB,sE,sNh,sLh,sE',sST} = \frac{0.01348\,\hat{m}^2_{sST^*}\,\widehat{pNh}_{sNh}\widehat{pLh}_{sLh}\widehat{pNE}_{sE}}{\hat{\rho}_{sST^*}\,\widehat{pdh}_{sd,sD}\,\widehat{pAa}^2_{sd,sD}\widehat{pNB}^2_{sB}\widehat{pNE}^2_{sE'}} + \frac{16.328\,\hat{\mu}^{0.93}_{sST^*}\,\hat{m}^{1.07}_{sST^*}\,\widehat{pNh}_{sNh}\widehat{pLh}_{sLh}\widehat{pNE}_{sE}}{\hat{\rho}_{sST^*}\,\widehat{pdh}^{1.93}_{sd,sD}\,\widehat{pAa}^{1.07}_{sd,sD}\widehat{pNB}^{1.07}_{sB}\widehat{pNE}^{1.07}_{sE'}} \tag{73}$$

$$\widehat{pdPa}4_{sd,sD,sB,sE,sNh,sLh,sE',sST} = \frac{0.089\;\hat{\mu}^{0.1865}_{sST^*}\,\hat{m}^{1.8135}_{sST^*}\,\widehat{pNh}_{sNh}\widehat{pLh}_{sLh}\widehat{pNE}_{sE}}{\hat{\rho}_{sST^*}\,\widehat{pdh}^{1.1865}_{sd,sD}\,\widehat{pAa}^{1.8135}_{sd,sD}\widehat{pNB}^{1.8135}_{sB}\widehat{pNE}^{1.8135}_{sE'}} \tag{74}$$

where if $sST = h$, then $sST^{*} = c$ and vice-versa.

3.6. Heat transfer rate equation

$$A = \sum_{sd=1}^{sdmax} \sum_{sB=1}^{sBmax} \sum_{sNh=1}^{sNhmax} \sum_{sLh=1}^{sLhmax} \sum_{sE=1}^{sEmax} \sum_{sE'=1}^{sE'max} \widehat{pA}_{sd,sB,sE,sNh,sLh,sE'}\, wA_{sd,sB,sE,sNh,sLh,sE'} \quad (75)$$

$$wA_{sd,sB,sNh,sLh,sE,sE'} \leq yd_{sd} \quad (76)$$

$$wA_{sd,sB,sNh,sLh,sE,sE'} \leq yB_{sB} \quad (77)$$

$$wA_{sd,sB,sNh,sLh,sE,sE'} \leq yNh_{sNh} \quad (78)$$

$$wA_{sd,sB,sNh,sLh,sE,sE'} \leq yLh_{sLh} \quad (79)$$

$$wA_{sd,sB,sNh,sLh,sE,sE'} \leq yPt_{sE} \quad (80)$$

$$wA_{sd,sB,sNh,sLh,sE,sE'} \leq yPa_{sE'} \quad (81)$$

$$wA_{sd,sB,sNh,sLh,sE,sE'} \geq yd_{sd} + yB_{sB} + yNh_{sNh} + yLh_{sLh} + yPt_{sE} + yPa_{sE'} - 5 \quad (82)$$

$$\widehat{pA}_{sd,sB,sNh,sLh,sE,sE'} = \pi\ \widehat{pNB}_{sB}\ \widehat{pNh}_{sNh}\widehat{pLh}_{sLh}\widehat{pNE}_{sE}\widehat{pNE}_{sE'} \quad (83)$$

The heat transfer rate, based on the LMTD method, after reformulation becomes:

$$\hat{Q}\sum_{sd=1}^{sdmax}\sum_{sD=1}^{sDmax}\sum_{sB=1}^{sBmax}\sum_{sE=1}^{sEmax}\sum_{sLh=1}^{sLhmax}\sum_{sE'=1}^{sEmax}\sum_{sST}\left[\sum_{sRet=1}^{2}\left(\frac{\widehat{pdte}_{sd}}{\hat{k}_{sST}\,\widehat{pNut}^{theo}}wht_{sd,sST,sRet,1,1}^{theo}\right.\right.$$

$$+\frac{\widehat{pdte}_{sd}}{\hat{k}_{sST}\,\widehat{pNut}_{sB,sE,sLh,sST}^{S\&T}}wht_{sd,sB,sLh,sE,sST,sRet,1,2}^{S\&T}+\frac{\widehat{pdte}_{sd}}{\hat{k}_{sST}\,\widehat{pNut}_{sB,sE,sLh,sST}^{Hau}}wht_{sd,sB,sLh,sE,sST,sRet,2}^{Hau}\Bigg)$$

$$+\frac{\widehat{pdte}_{sd}}{\hat{k}_{sST}\,\widehat{pNut}_{tran_{sd,sB,sE,sST}}^{Gni}}wht_{sd,sB,sE,sST,3}^{Gni}+\frac{\widehat{pdte}_{sd}}{\hat{k}_{sST}\,\widehat{pNut}_{turb_{sd,sB,sE,sST}}^{Gni}}wht_{sd,sB,sE,sST,4}^{Gni}$$

$$+\widehat{Rf}_{sST}\frac{\widehat{pdte}_{sd}}{\widehat{pdti}_{sd}}wydT_{sd,sST}+\frac{\widehat{pdte}_{sd}\ln\left(\frac{\widehat{pdte}_{sd}}{\widehat{pdti}_{sd}}\right)}{2\widehat{ktube}}yd_{sd}+\widehat{Rf}_{sST^*}yT_{sST}$$

$$+\sum_{sRea=1}^{2}\left(\frac{\widehat{pdh}_{sd,sD}}{\hat{k}_{sST^*}\,\widehat{pNua}^{theo}}wha_{sd,sD,sST,sRea,1,1}^{theo}\right.$$

$$+\frac{\widehat{pdh}_{sd,sD}}{\hat{k}_{sST^*}\,\widehat{pNua}_{sd,sD,sB,sE',sLh,sST}^{S\&T}}wha_{sd,sD,sB,sE',sLh,sST,sRea,1,2}^{S\&T} \tag{84}$$

$$\left.+\frac{\widehat{pdh}_{sd,sD}}{\hat{k}_{sST^*}\,\widehat{pNua}_{sd,sD,sB,sE',sLh,sST}^{Hau}}wha_{sd,sD,sB,sLh,sE',sST,sRea,2}^{Hau}\right)$$

$$\left.+\frac{\widehat{pdh}_{sd,sD}}{\hat{k}_{sST^*}\,\widehat{pNua}_{tran_{sd,sD,sB,sE',sST}}^{Gni}}wha_{sd,sD,sB,sE',sST,3}^{Gni}+\frac{\widehat{pdh}_{sd,sD}}{\hat{k}_{sST^*}\,\widehat{pNua}_{turb_{sd,sD,sB,sE',sST}}^{Gni}}wha_{sd,sD,sB,sE',sST,4}^{Gni}\right]$$

$$\leq\frac{\widehat{\Delta Tlm}}{\left(\frac{\widehat{Aexc}}{100}+1\right)}\sum_{sd=1}^{sdmax}\sum_{sB=1}^{sBmax}\sum_{sE=1}^{sEmax}\sum_{sNh=1}^{sNhmax}\sum_{sLh=1}^{sLhmax}\sum_{sE'=1}^{sE'max}\sum_{sST}(\widehat{pA}_{sd,sB,sNh,sLh,sE,sE'}\,wA_{sd,sB,sNh,sLh,sE}$$

$$+\widehat{pA}_{sd,sB,sNh,sLh,sE,sE'}\left(\widehat{pF}_{sST^*,sE}-1\right)wAF_{sd,sB,sNh,sLh,sE,sE',sST}$$

$$+\widehat{pA}_{sd,sB,sNh,sLh,sE,sE'}\left(\widehat{pF}_{sST,sE'}-1\right)wAF_{sd,sB,sNh,sLh,sE,sE',sST})$$

$$wydT_{sd,sST}\leq yd_{sd} \tag{85}$$

$$wydT_{sd,sST}\leq yT_{sST} \tag{86}$$

$$wydT_{sd,sST}\geq yd_{sd}+yT_{sST}-1 \tag{87}$$

$$wht_{sd,sST,sRet,1,1}^{theo}\leq wydT_{sd,sST}\qquad \text{for } sRet=\{1,2\} \tag{88}$$

$$wht_{sd,sST,sRet,1,1}^{theo}\leq yRet_{sRet}\qquad \text{for } sRet=\{1,2\} \tag{89}$$

$$wht_{sd,sST,sRet,1,1}^{theo}\leq yPrt_{1}\qquad \text{for } sRet=\{1,2\} \tag{90}$$

$$wht_{sd,sST,sRet,1,1}^{theo}\leq yNut_{1}\qquad \text{for } sRet=\{1,2\} \tag{91}$$

$$wht^{theo}_{sd,sST,sRet,1,1} \geq wydT_{sd,sST} + yRet_{sRet} + yPrt_1 + yNut_1 - 3 \quad \text{for } sRet = \{1,2\} \quad (92)$$

$$wht^{S\&T}_{sd,sB,sE,sLh,sST,sRet,1,2} \leq wvt_{sd,sB,sE,sST} \quad \text{for } sRet = \{1,2\} \quad (93)$$

$$wht^{S\&T}_{sd,sB,sE,sLh,sST,sRet,1,2} \leq yLh_{sLh} \quad \text{for } sRet = \{1,2\} \quad (94)$$

$$wht^{S\&T}_{sd,sB,sE,sLh,sST,sRet,1,2} \leq yRet_{sRet} \quad \text{for } sRet = \{1,2\} \quad (95)$$

$$wht^{S\&T}_{sd,sB,sE,sLh,sST,sRet,1,2} \leq yPrt_1 \quad \text{for } sRet = \{1,2\} \quad (96)$$

$$wht^{S\&T}_{sd,sB,sE,sLh,sST,sRet,1,2} \leq yNut_2 \quad \text{for } sRet = \{1,2\} \quad (97)$$

$$wht^{S\&T}_{sd,sB,sE,sLh,sST,sRet,1,2} \geq wvt_{sd,sB,sE,sST} + yLh_{sLh} + yRet_{sRet} + yPrt_1 + yNut_2 - 4 \quad \text{for } sRet = \{1,2\} \quad (98)$$

$$wht^{Hau}_{sd,sB,sE,sLh,sST,sRet,2} \leq wvt_{sd,sB,sE,sST} \quad \text{for } sRet = \{1,2\} \quad (99)$$

$$wht^{Hau}_{sd,sB,sE,sLh,sST,sRet,2} \leq yLh_{sLh} \quad \text{for } sRet = \{1,2\} \quad (100)$$

$$wht^{Hau}_{sd,sB,sE,sLh,sST,sRet,2} \leq yRet_{sRet} \quad \text{for } sRet = \{1,2\} \quad (101)$$

$$wht^{Hau}_{sd,sB,sE,sLh,sST,sRet,2} \leq yPrt_2 \quad \text{for } sRet = \{1,2\} \quad (102)$$

$$wht^{Hau}_{sd,sB,sE,sLh,sST,sRet,2} \geq wvt_{sd,sB,sE,sST} + yLh_{sLh} + yRet_{sRet} + yPrt_2 - 3 \quad \text{for } sRet = \{1,2\} \quad (103)$$

$$wht^{Gni}_{sd,sB,sE,sST,sRet} \leq wvt_{sd,sB,sE,sST} \quad \text{for } sRet = \{3,4\} \quad (104)$$

$$wht^{Gni}_{sd,sB,sE,sST,sRet} \leq yRet_{sRet} \quad \text{for } sRet = \{3,4\} \quad (105)$$

$$wht^{Gni}_{sd,sB,sE,sST,sRet} \geq wvt_{sd,sB,sE,sST} + yRet_{sRet} - 1 \quad \text{for } sRet = \{3,4\} \quad (106)$$

$$wha^{theo}_{sd,sD,sST,sRea,1,1} \leq wydT_{sd,sST} \quad \text{for } sRea = \{1,2\} \quad (107)$$

$$wha^{theo}_{sd,sD,sST,sRea,1,1} \leq yD_{sD} \quad \text{for } sRea = \{1,2\} \quad (108)$$

$$wha^{theo}_{sd,sD,sST,sRea,1,1} \leq yRea_{sRea} \quad \text{for } sRea = \{1,2\} \quad (109)$$

$$wha^{theo}_{sd,sD,sST,sRea,1,1} \leq yPra_1 \qquad \text{for } sRea = \{1,2\} \tag{110}$$

$$wha^{theo}_{sd,sD,sST,sRea,1,1} \leq yNua_1 \qquad \text{for } sRea = \{1,2\} \tag{111}$$

$$wha^{theo}_{sd,sD,sST,sRea,1,1} \geq wydT_{sd,sST} + yD_{sD} + yRea_{sRea} + yPra_1 + yNua_1 - 4 \qquad \text{for } sRea = \{1,2\} \tag{112}$$

$$wha^{S\&T}_{sd,sD,sB,sE',sLh,sST,sRea,1,2} \leq wva_{sd,sD,sB,sE',sST} \qquad \text{for } sRea = \{1,2\} \tag{113}$$

$$wha^{S\&T}_{sd,sD,sB,sE',sLh,sST,sRea,1,2} \leq yLh_{sLh} \qquad \text{for } sRea = \{1,2\} \tag{114}$$

$$wha^{S\&T}_{sd,sD,sB,sE',sLh,sST,sRea,1,2} \leq yRea_{sRea} \qquad \text{for } sRea = \{1,2\} \tag{115}$$

$$wha^{S\&T}_{sd,sD,sB,sE',sLh,sST,sRea,1,2} \leq yPra_1 \qquad \text{for } sRea = \{1,2\} \tag{116}$$

$$wha^{S\&T}_{sd,sD,sB,sE',sLh,sST,sRea,1,2} \leq yNua_2 \qquad \text{for } sRea = \{1,2\} \tag{117}$$

$$wha^{S\&T}_{sd,sD,sB,sE',sLh,sST,sRea,1,2} \geq va_{sd,sD,sB,sE',sST} + yLh_{sLh} + yRea_{sRea} + yPra_1 + yNua_2 - 4 \qquad \text{for } sRea = \{1,2\} \tag{118}$$

$$wha^{Hau}_{sd,sD,sB,sE',sLh,sST,sRea,2} \leq wva_{sd,sD,sB,sE',sST} \qquad \text{for } sRea = \{1,2\} \tag{119}$$

$$wha^{Hau}_{sd,sD,sB,sE',sLh,sST,sRea,2} \leq yLh_{sLh} \qquad \text{for } sRea = \{1,2\} \tag{120}$$

$$wha^{Hau}_{sd,sD,sB,sE',sLh,sST,sRea,2} \leq yRea_{sRea} \qquad \text{for } sRea = \{1,2\} \tag{121}$$

$$wha^{Hau}_{sd,sD,sB,sE',sLh,sST,sRea,2} \leq yPra_2 \qquad \text{for } sRea = \{1,2\} \tag{122}$$

$$wha^{Hau}_{sd,sD,sB,sE',sLh,sST,sRea,2} \geq wva_{sd,sD,sB,sE',sST} + yLh_{sLh} + yRea_{sRea} + yPra_2 - 3 \qquad \text{for } sRea = \{1,2\} \tag{123}$$

$$wha^{Gni}_{sd,sD,sB,sE',sST,sRea} \leq wva_{sd,sD,sB,sE',sST} \qquad \text{for } sRea = \{3,4\} \tag{124}$$

$$wha^{Gni}_{sd,sD,sB,sE',sST,sRea} \leq yRea_{sRea} \qquad \text{for } sRea = \{3,4\} \tag{125}$$

$$wha^{Gni}_{sd,sD,sB,sE',sST,sRea}$$

$$\geq wva_{sd,sD,sB,sE\prime,sST} + yRea_{sRea} \tag{126}$$

$$-1 \qquad \text{for } sRea = \{3,4\}$$

$$wAF_{sd,sB,sNh,sLh,sE,sE\prime,sST} \leq wA_{sd,sB,sNh,sLh,sE,sE'} \tag{127}$$

$$wAF_{sd,sB,sNh,sLh,sE,sE\prime,sST} \leq yT_{sST} \tag{128}$$

$$wAF_{sd,sB,sNh,sLh,sE,sE\prime,sST} \geq wA_{sd,sB,sNh,sLh,sE,sE'} + yT_{sST} - 1 \tag{129}$$

where

$$\widehat{pNut}^{Hau}_{sB,sE,sLh,sST} = 3.66 + \frac{0{,}0668\left(\frac{8\,\widehat{Cp}_{sST}\,\widehat{m}_{sST}}{\pi\,\hat{k}_{sST}\,\widehat{pNB}_{sB}\,\widehat{pNE}_{sE}\,\widehat{pLh}_{sLh}}\right)}{1 + 0{,}04\left(\frac{8\,\widehat{Cp}_{sST}\,\widehat{m}_{sST}}{\pi\,\hat{k}_{sST}\,\widehat{pNB}_{sB}\,\widehat{pNE}_{sE}\,\widehat{pLh}_{sLh}}\right)^{2/3}} \tag{130}$$

$$\widehat{pNut}^{Gni}_{tran_{sd,sB,sE,sST}} = \frac{0.0061\left(\frac{4\;\widehat{m}_{sST}}{\pi\,\hat{\mu}_{sST}\,\widehat{pdti}_{sd}\,\widehat{pNB}_{sB}\,\widehat{pNE}_{sE}} - 1000\right)\left(\frac{\widehat{Cp}_{sST}\,\hat{\mu}_{sST}}{\hat{k}_{sST}}\right)}{1 + 12.7(0.0061)^{\frac{1}{2}}\left(\left(\frac{\widehat{Cp}_{sST}\,\hat{\mu}_{sST}}{\hat{k}_{sST}}\right)^{\frac{2}{3}} - 1\right)} \tag{131}$$

$$\widehat{pNut}^{Gni}_{turb_{sd,sB,sE,sST}}$$

$$= \frac{\left(0.00175 + 0.132\left(\frac{\pi\,\hat{\mu}_{sST}\,\widehat{pdti}_{sd}\,\widehat{pNB}_{sB}\,\widehat{pNE}_{sE}}{4\;\widehat{m}_{sST}}\right)^{0.42}\right)\left(\frac{4\;\widehat{m}_{sST}}{\pi\,\hat{\mu}_{sST}\,\widehat{pdti}_{sd}\,\widehat{pNB}_{sB}\,\widehat{pNE}_{sE}} - \right.}{1 + 12.7\left(0.00175 + 0.132\left(\frac{\pi\,\hat{\mu}_{sST}\,\widehat{pdti}_{sd}\,\widehat{pNB}_{sB}\,\widehat{pNE}_{sE}}{4\;\widehat{m}_{sST}}\right)^{0.42}\right)^{1/2}\left(\left(\frac{\widehat{Cp}_{sST}\,\hat{\mu}}{\hat{k}_{sST}}\right.\right.} \tag{132}$$

$$\widehat{pNua}^{Hau}_{sd,sD,sB,sE,sLh,sST} = 3.66 + \frac{0.0668\;\frac{2\,\widehat{Cp}_{sST^*}\,\widehat{m}_{sST^*}\,\widehat{pdh}^2_{sd,sD}}{\hat{k}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}_{sB}\widehat{pNE}_{sE}\widehat{pLh}_{sLh}}}{1 + 0.04\left(\frac{2\,\widehat{Cp}_{sST^*}\,\widehat{m}_{sST^*}\,\widehat{pdh}^2_{sd,sD}}{\hat{k}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}_{sB}\widehat{pNE}_{sE}\widehat{pLh}_{sLh}}\right)^{\frac{2}{3}}} \tag{133}$$

$$\widehat{pNua}^{Gni}_{tran_{sd,sD,sB,sE,sST}} = \frac{\left(0.00337 + 4.082\left(\frac{\hat{\mu}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}{\hat{m}_{sST^*}\,\widehat{pdh}_{sd,sD}}\right)^{0.93}\right)\left(\frac{\hat{m}_{sST^*}\,\widehat{pdh}_{sd,sD}}{\hat{\mu}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}_{sB}\widehat{pNE}_{sE}} - \right.}{1 + 12.7\left(0.00337 + 4.082\left(\frac{\hat{\mu}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}{\hat{m}_{sST^*}\,\widehat{pdh}_{sd,sD}}\right)^{0.93}\right)^{\frac{1}{2}}\left(\left(\frac{\widehat{Cp}_{sST^*}\hat{\mu}_{sST}}{\hat{k}_{sST^*}}\right.\right.} \quad (134)$$

$$\widehat{pNua}^{Gni}_{turb_{sd,sD,sB,sE,sST}} = \frac{\left(0.02225\left(\frac{\hat{\mu}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}{\hat{m}_{sST^*}\,\widehat{pdh}_{sd,sD}}\right)^{0.1865}\right)\left(\frac{\hat{m}_{sST^*}\,\widehat{pdh}_{sd,sD}}{\hat{\mu}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}_{sB}\widehat{pNE}_{sE}} - 1000\right)}{1 + 12.7\left(0.02225\left(\frac{\hat{\mu}_{sST^*}\,\widehat{pAa}_{sd,sD}\widehat{pNB}_{sB}\widehat{pNE}_{sE}}{\hat{m}_{sST^*}\,\widehat{pdh}_{sd,sD}}\right)^{0.1865}\right)^{\frac{1}{2}}\left(\left(\frac{\widehat{Cp}_{sST^*}\hat{\mu}_{sST^*}}{\hat{k}_{sST^*}}\right)^{\frac{2}{3}} - \right.} \quad (135)$$

where if $sST = h$, then $sST^* = c$ and vice-versa.

3.7. *Pressure drop and velocity bounds.*

Bounds on flow velocity and pressure drop are given by:

$$vt \geq \widehat{vt}_{min} \quad (136)$$

$$vt \leq \widehat{vt}_{max} \quad (137)$$

$$va \geq \widehat{va}_{min} \quad (138)$$

$$va \leq \widehat{va}_{max} \quad (139)$$

$$\Delta Pt \leq \widehat{\Delta P}_{c_{disp}} yT_c + \widehat{\Delta P}_{h_{disp}} yT_h \quad (140)$$

$$\Delta Pa \leq \widehat{\Delta P}_{c_{disp}} yT_h + \widehat{\Delta P}_{h_{disp}} yT_c \quad (141)$$

3.8. *Objective function*

The objective function is given by the heat transfer area and a penalty term associated to the number of hairpins; thus, in case of equivalent solutions, the one with smaller number of elements is preferred:

$$\min A + \sum_{sNh=1}^{sNhmax} \widehat{ph}\ \widehat{pNh}_{sNh} yNh_{sNh} \quad (142)$$